Lecture 11

Reflection and Refraction

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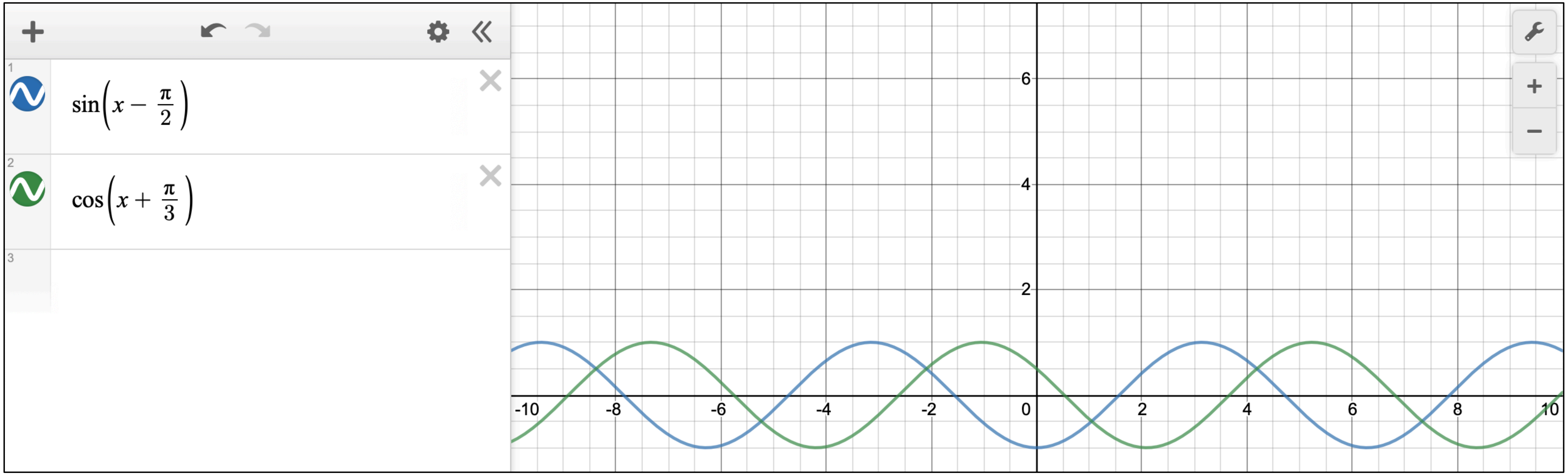
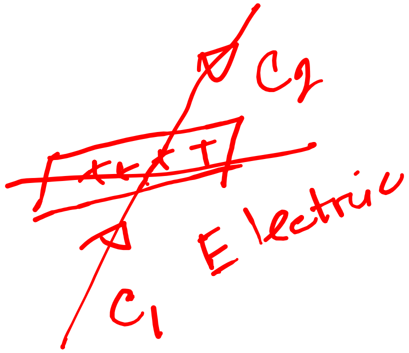
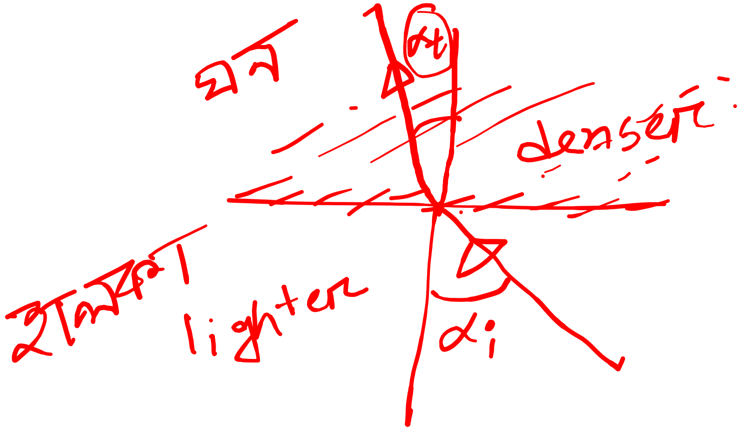
Sin and Cosine Graph Visualization

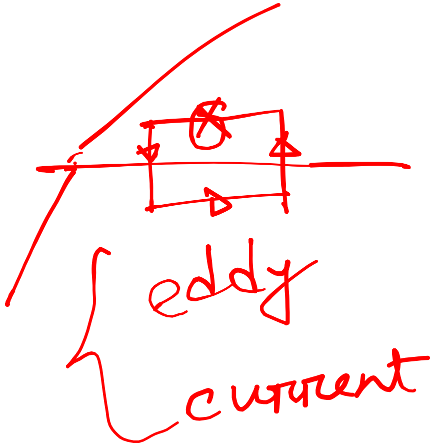
- https://www.desmos.com/calculator







$(\alpha_t < \alpha_i)$



Reflection and Refraction

(Diffraction and scattering does not occur at plane boundaries of infinite extension. This happens at edges and corners or on rough surfaces.)

$$\Delta Q = \rho_S \Delta S = D_{1n}\Delta S - D_{2n}\Delta S$$

$$\boxed{D_{1n} - D_{2n} = \rho_S}$$

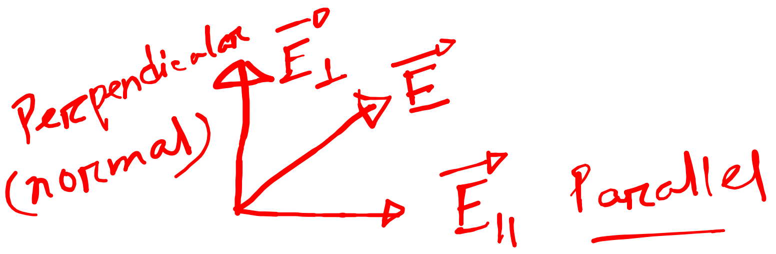
where ρ_S is the free charge density placed deliberately at the boundary. It should be borne in mind that eq. (5.59) is based on the assumption that **D** is directed from region 2 to region 1 and eq. (5.59) must be applied accordingly. If no free charges exist at the interface (i.e., charges are not deliberately placed there), $\rho_S = 0$ and eq. (5.59) becomes

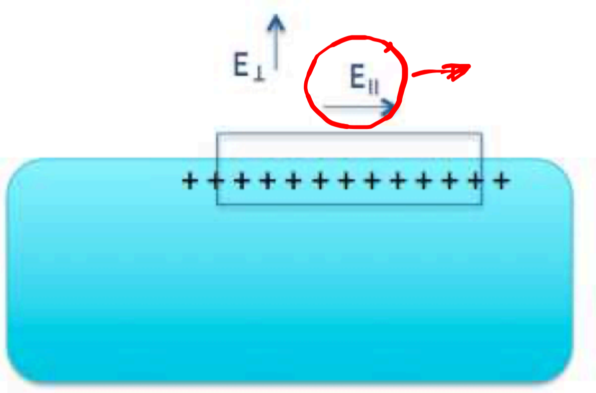
$$\boxed{D_{1n} = D_{2n}} \tag{5.60}$$

Thus the normal component of **D** is continuous across the interface; that is, D_n undergoes no change at the boundary. Since $\mathbf{D} = \varepsilon\mathbf{E}$, eq. (5.60) can be written as

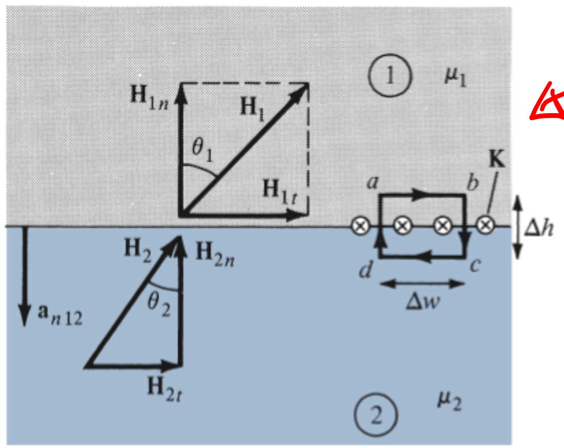
$$\varepsilon_1 E_{1n} = \varepsilon_2 E_{2n} \tag{5.61}$$

$(E_{1t} = E_{2t})$





Charge accumulating on a boundary can affect $E_\perp$ not $E_\parallel$.



(b)

As $\Delta h \rightarrow 0$, eq. (8.42) leads to

$$H_{1t} - H_{2t} = K \tag{8.43}$$

This shows that the tangential component of H is also discontinuous. Equation (8.43) may be written in terms of B as

$$\frac{B_{1t}}{\mu_1} - \frac{B_{2t}}{\mu_2} = K \tag{8.44}$$

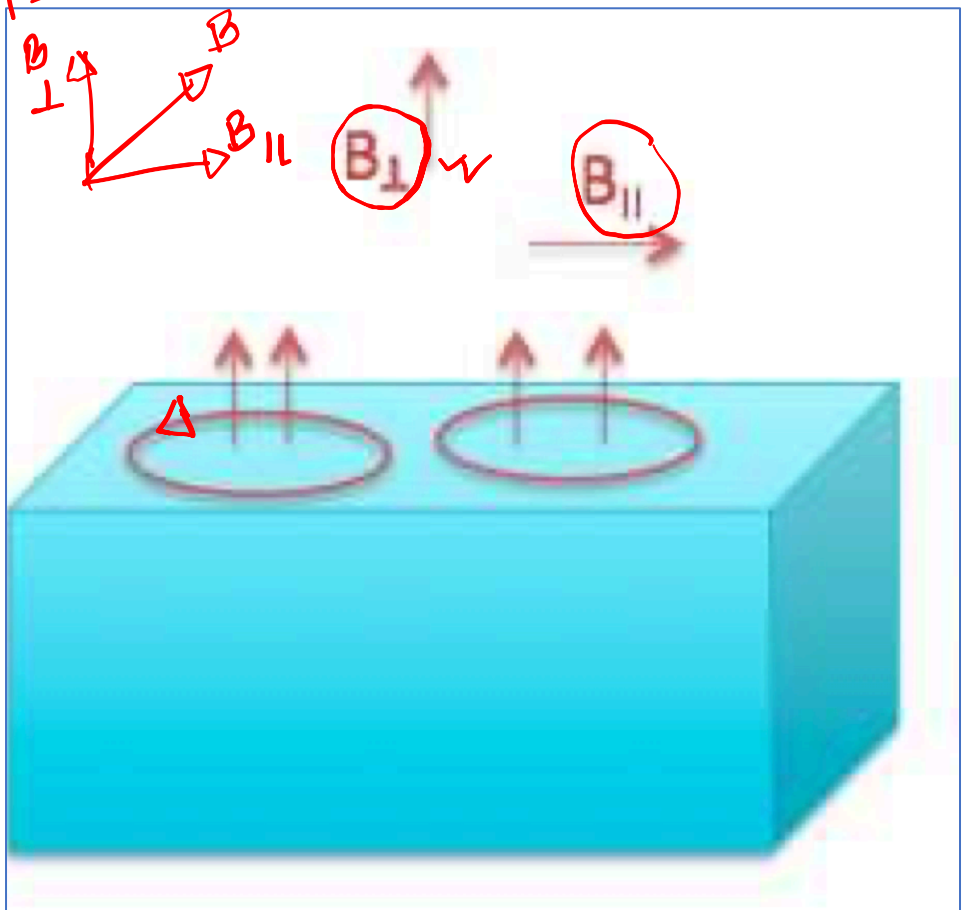
In the general case, eq. (8.43) becomes

$$\boxed{(\mathbf{H}_1 - \mathbf{H}_2) \times \mathbf{a}_{n12} = \mathbf{K}} \tag{8.45}$$

where $\mathbf{a}_{n12}$ is a unit vector normal to the interface and is directed from medium 1 to medium 2. If the boundary is free of current or the media are not conductors (for $\mathbf{K}$ is free current density), $\mathbf{K} = \mathbf{0}$ and eq. (8.43) becomes

$$\boxed{\mathbf{H}_{1t} = \mathbf{H}_{2t}} \quad \text{or} \quad \frac{\mathbf{B}_{1t}}{\mu_1} = \frac{\mathbf{B}_{2t}}{\mu_2} \tag{8.46}$$

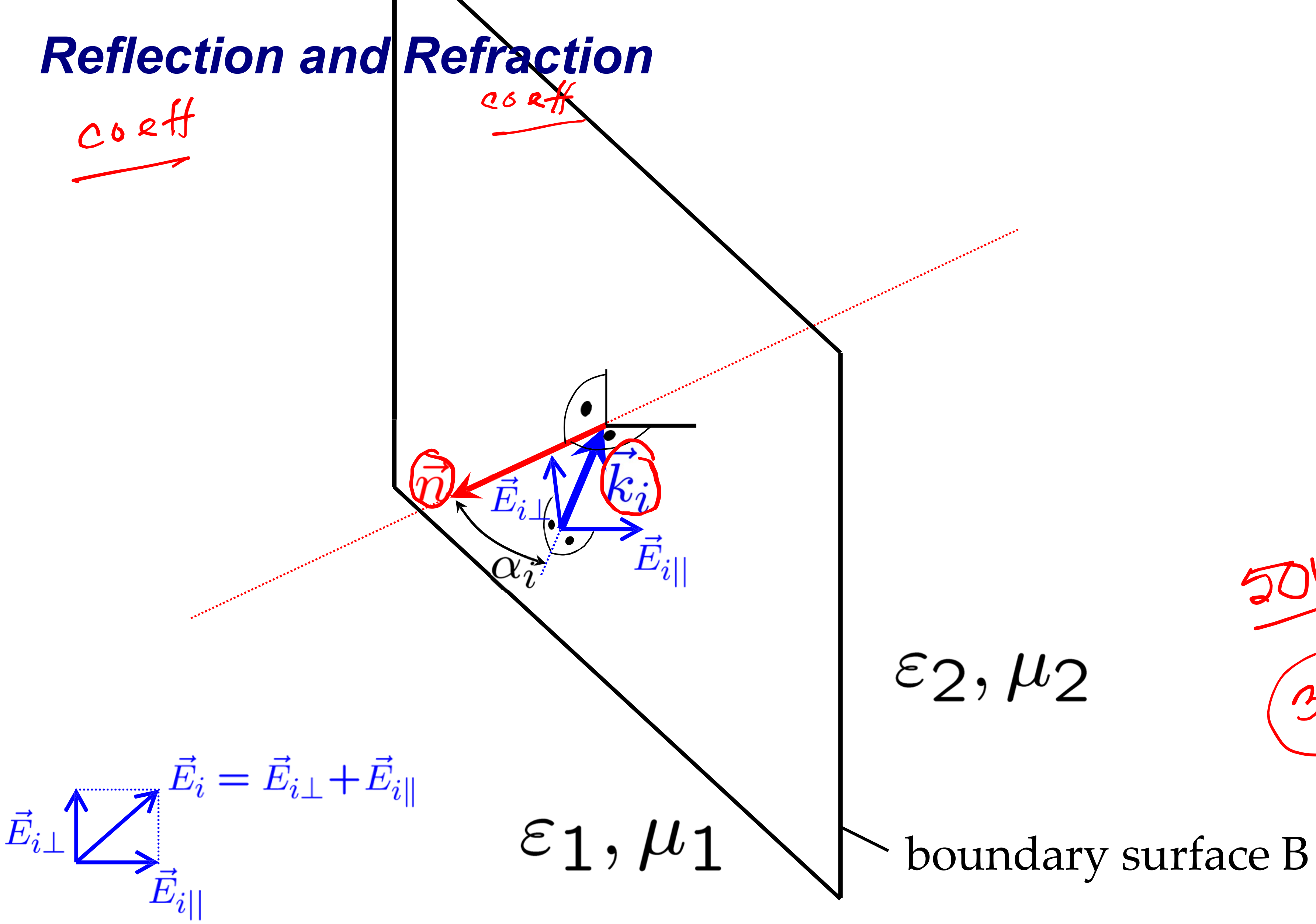
0



5. Current on the boundary can affect $B_{||}$ not $B_\perp$.

Reflection and Refraction

coeff



50W

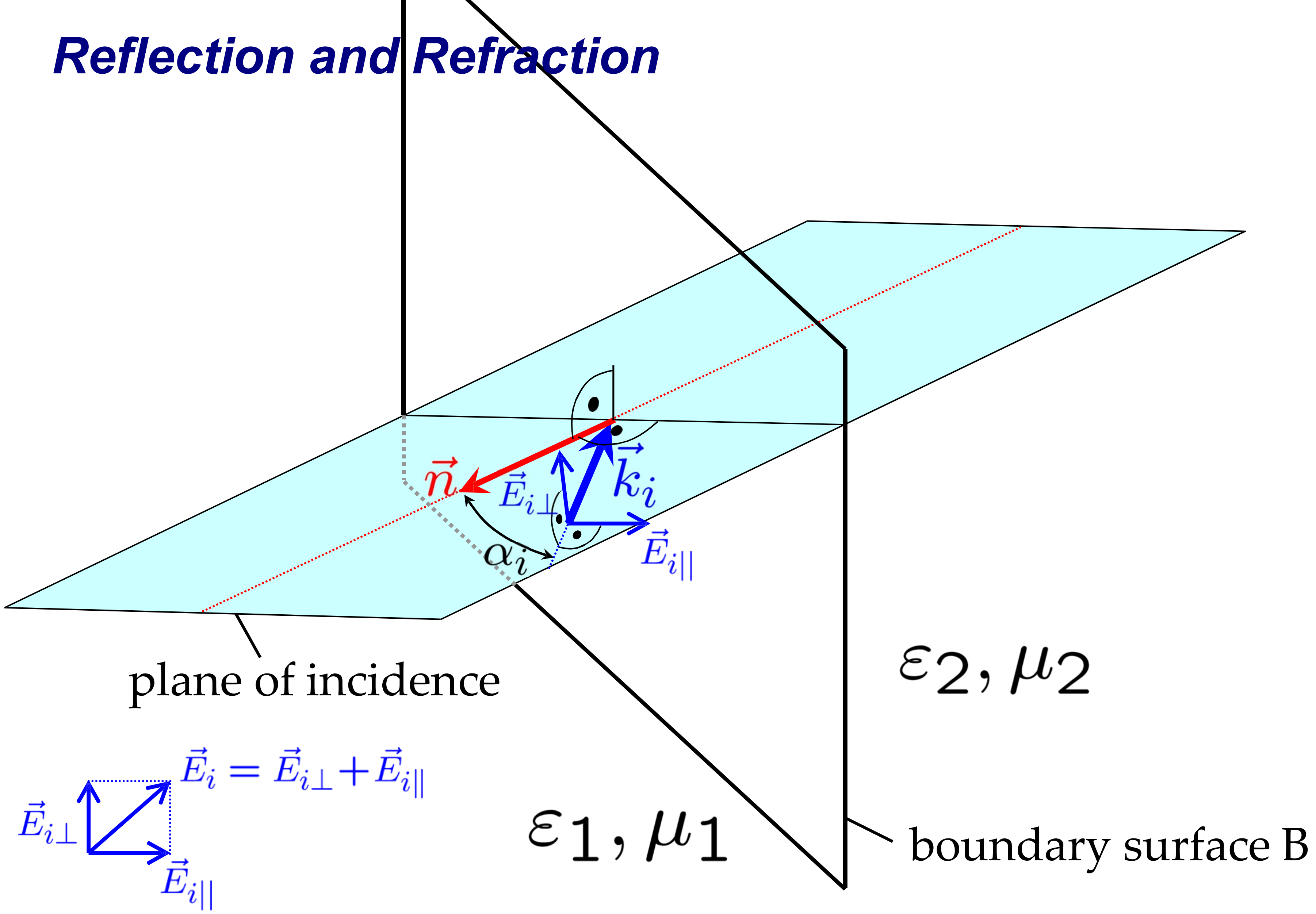
−20W

30W

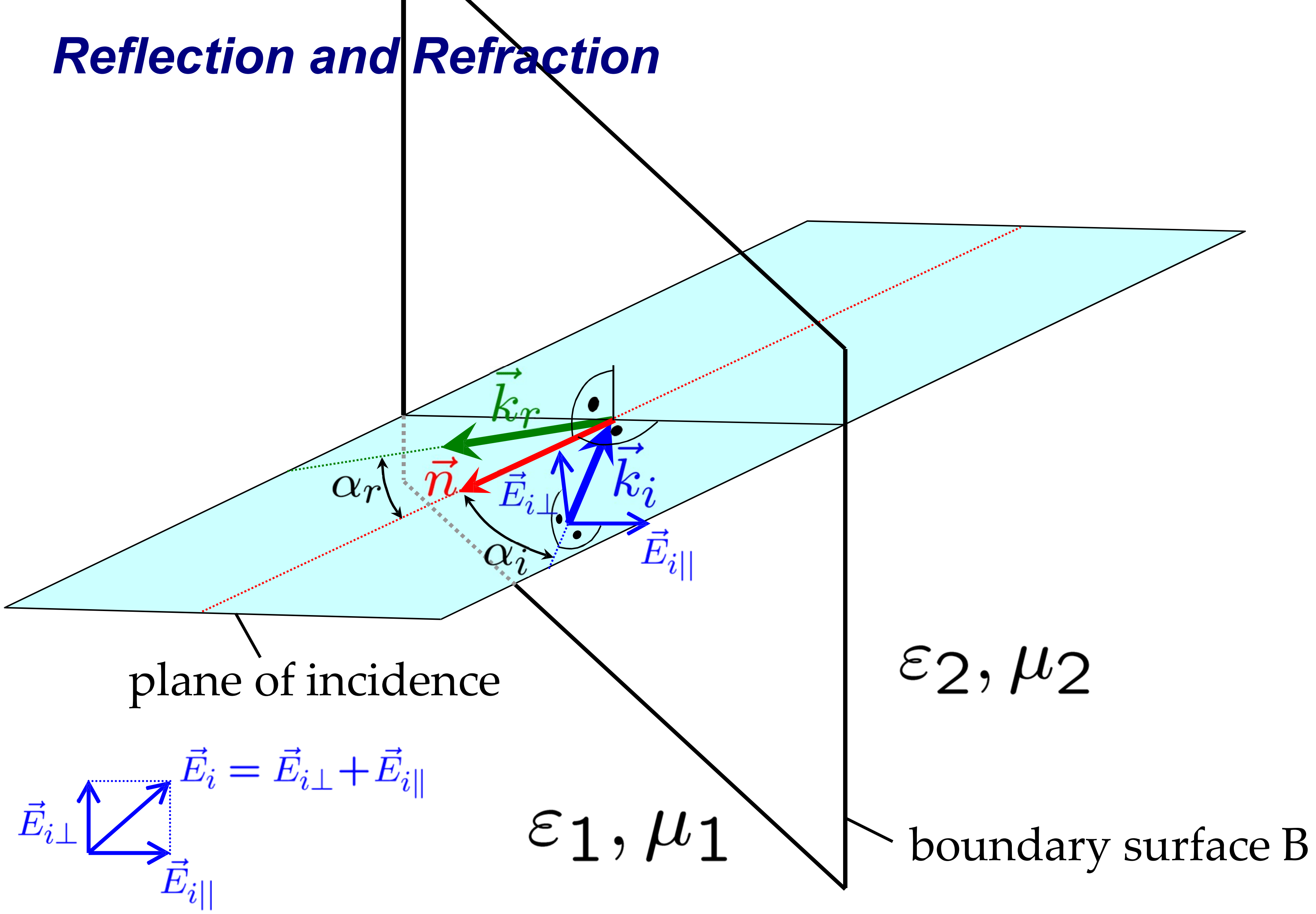
$r = 3/5$

$t = 2/5$

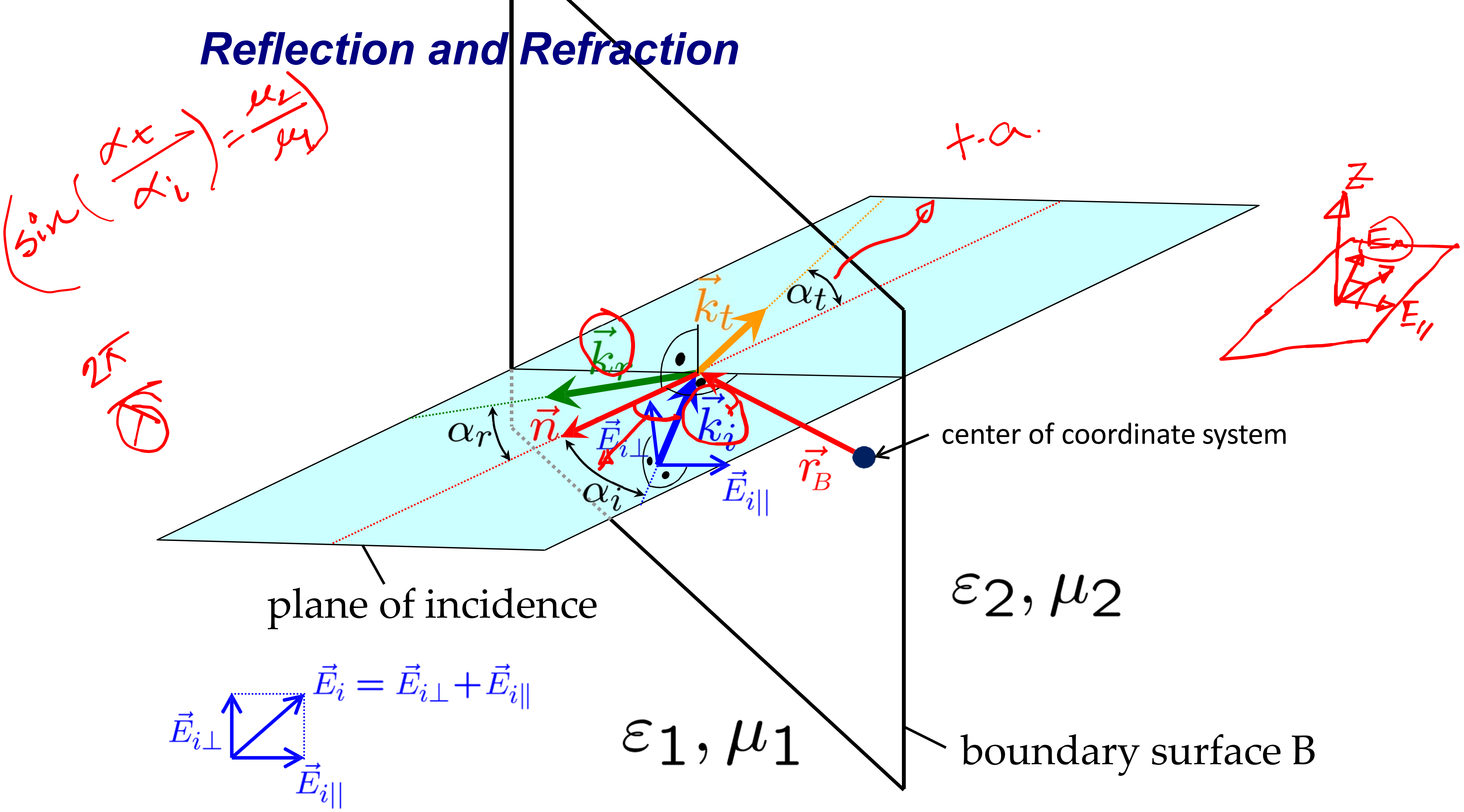
Reflection and Refraction



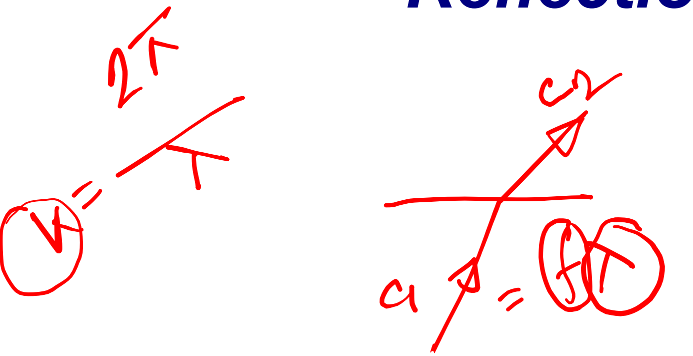
Reflection and Refraction



Reflection and Refraction



Reflection and Refraction



$$\boxed{\alpha_i = \alpha_r}$$

Law of Reflection

❖ The **law of reflection** states that when a ray of light reflects off a surface, the **angle of incidence is equal to the angle of reflection.**

$$\boxed{\frac{\sin \alpha_t}{\sin \alpha_i} = \frac{k_i}{k_t} = \frac{\sqrt{\mu_1 \varepsilon_1}}{\sqrt{\mu_2 \varepsilon_2}}}$$

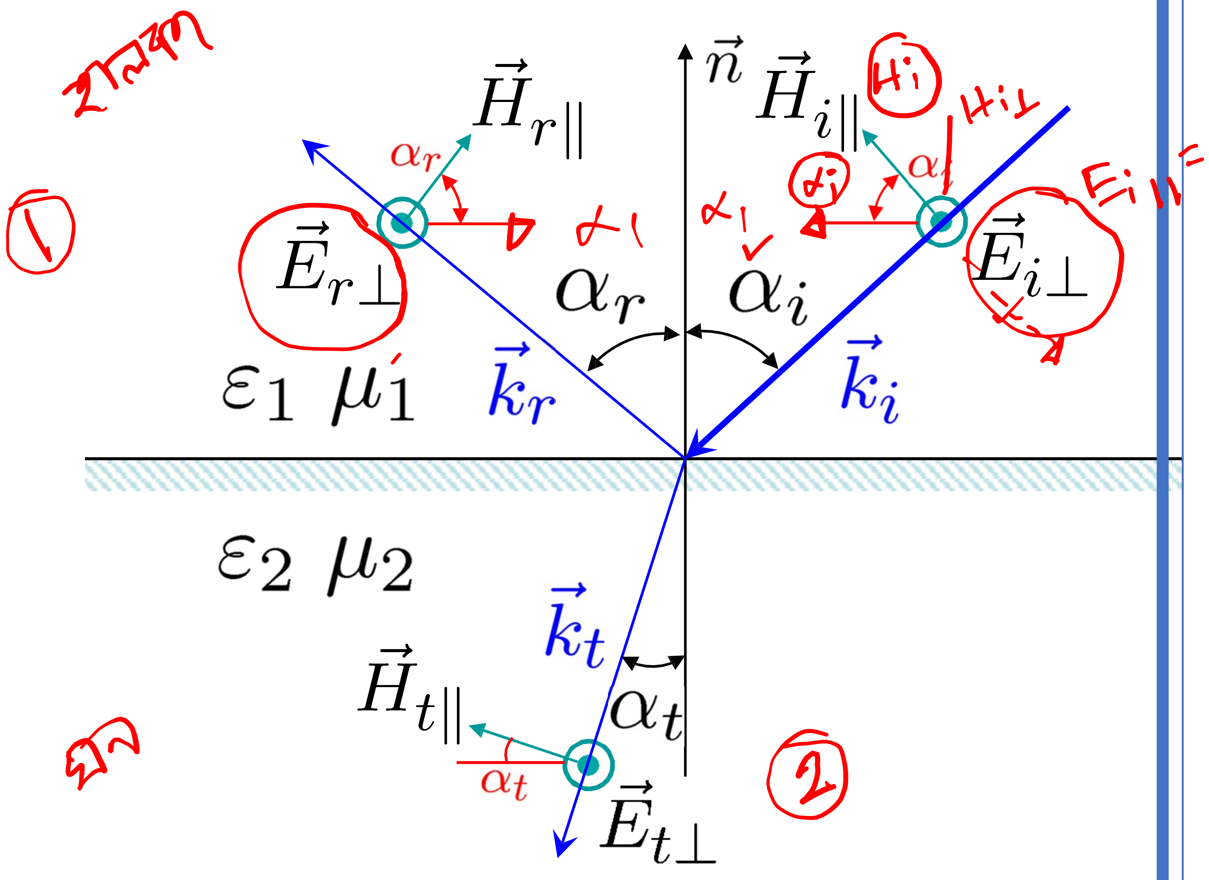
Snell's Law of Refraction

❖ Snell's law is a formula used to describe the relationship **between the angles of incidence and refraction**, when referring to light or other waves passing through a boundary between two different isotropic media, such as water, glass, or air.

Reflection and Refraction

TEM

Perpendicular Polarization



*** The electric field is perpendicular to the plane of incidence and the magnetic field is parallel to the plane of incidence.

$$\vec{E}_{1,tan} = \vec{E}_{2,tan}$$

$$\boxed{\vec{H}_{1,tan} = \vec{H}_{2,tan}}$$

and this can be written with incident, reflected and transmitted waves and angles $\alpha_i = \alpha_r = \alpha_1$, $\alpha_t = \alpha_2$ as

$$E_{i0\perp} + E_{r0\perp} = E_{t0\perp} \tag{4.96}$$

$$H_{i0||}\cos\alpha_1 - H_{r0||}\cos\alpha_1 = H_{t0||}\cos\alpha_2 \tag{4.97}$$

Please note that the directions of the field vectors can be arbitrarily chosen. It is only required that $\vec{E}\times\vec{H}$ points into the direction of propagation, i.e., the Poynting vector has be in parallel to the respective wave vector $\vec{k}$, and not anti-parallel.
With the wave impedances Z_1 and Z_2 ($Z_1 = \sqrt{\mu_1/\varepsilon_1}$, $Z_2 = \sqrt{\mu_2/\varepsilon_2}$), eq. (4.97) can be written as

$$(E_{i0\perp} - E_{r0\perp})\frac{\cos\alpha_1}{Z_1} = E_{t0\perp}\frac{\cos\alpha_2}{Z_2} \tag{4.98}$$

By inserting eq. (4.96) into (4.98) we get

$$(E_{i0\perp} - E_{r0\perp})\frac{\cos\alpha_1}{Z_1} = (E_{i0\perp} + E_{r0\perp})\frac{\cos\alpha_2}{Z_2} \tag{4.99}$$

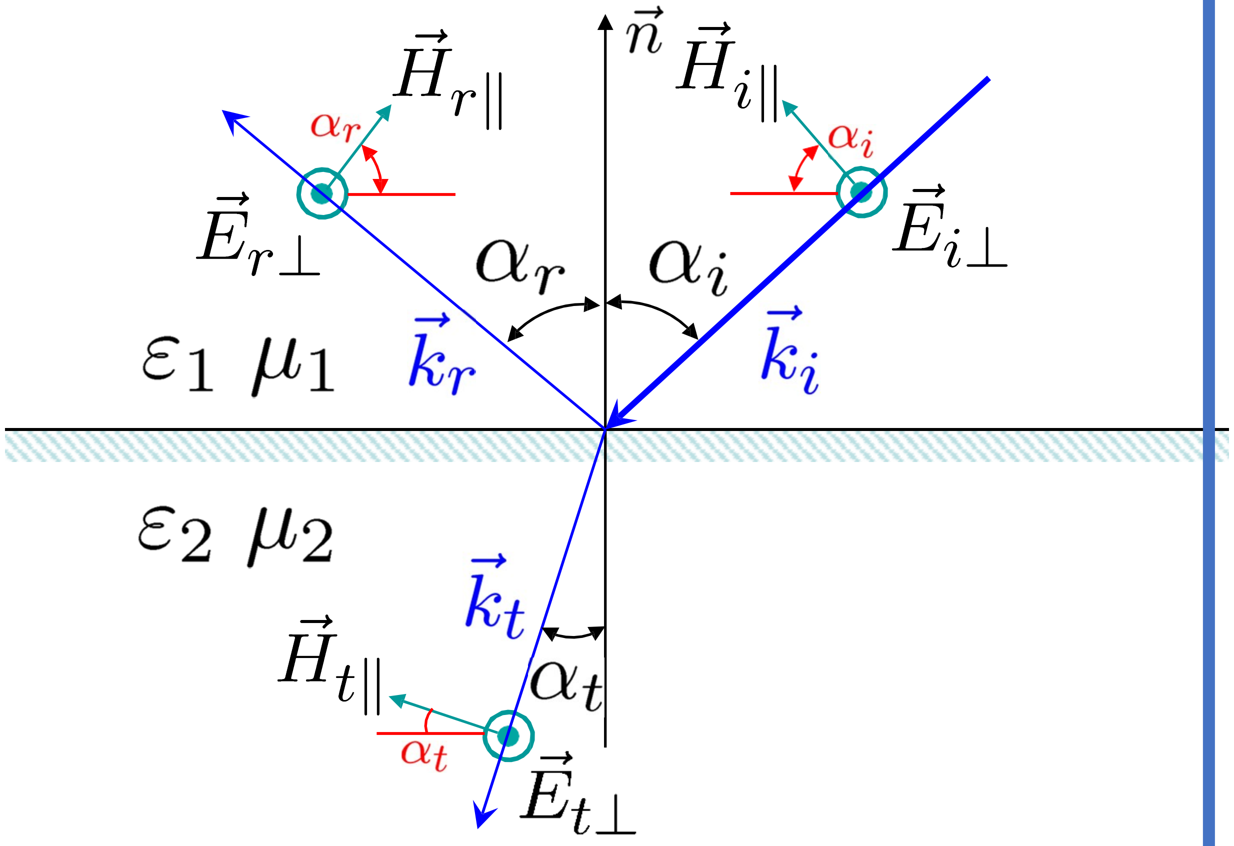
From this relation we can derive the reflection coefficient of the incident wave for perpendicular polarization:

$$\boxed{r_\perp = \frac{E_{r0\perp}}{E_{i0\perp}} = \frac{Z_2\cos\alpha_1 - Z_1\cos\alpha_2}{Z_2\cos\alpha_1 + Z_1\cos\alpha_2}} \tag{4.100}$$

Reflection and Refraction

$Z_2 = \sqrt{\mu_2/\epsilon_2}$

Perpendicular Polarization



*** The electric field is perpendicular to the plane of incidence and the magnetic field is parallel to the plane of incidence.

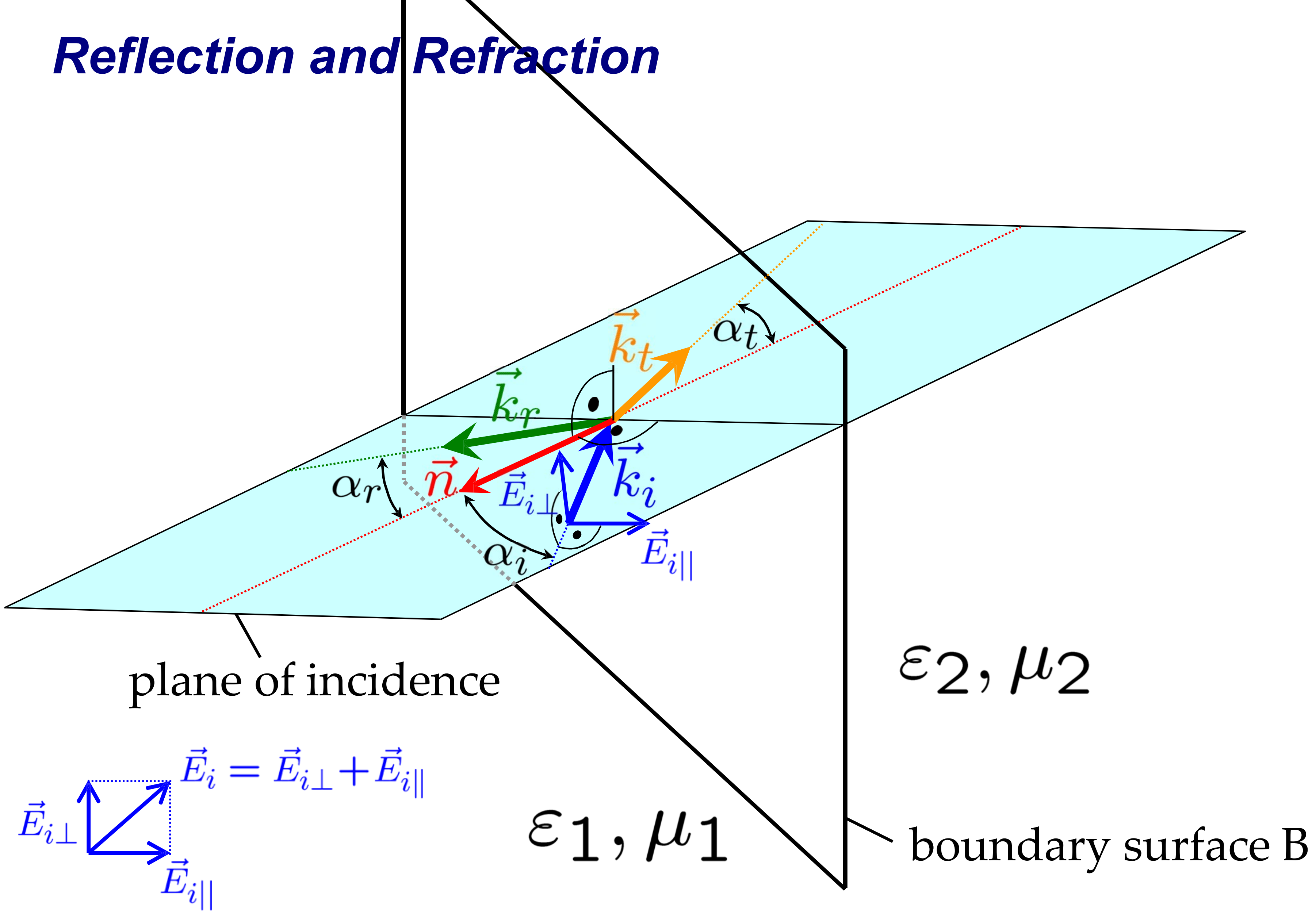
Solving the boundary conditions yields the reflection and transmission coefficients:

$$\boxed{r_\perp = \frac{E_{r0\perp}}{E_{i0\perp}} = \frac{Z_2\cos\alpha_1 - Z_1\cos\alpha_2}{Z_2\cos\alpha_1 + Z_1\cos\alpha_2}}$$ ✓ **perpendicular polarization**

$$\boxed{t_\perp = \frac{2\sqrt{Z_1 Z_2 \cos\alpha_1 \cos\alpha_2}}{Z_2\cos\alpha_1 + Z_1\cos\alpha_2}}$$ ✓ **perpendicular polarization**

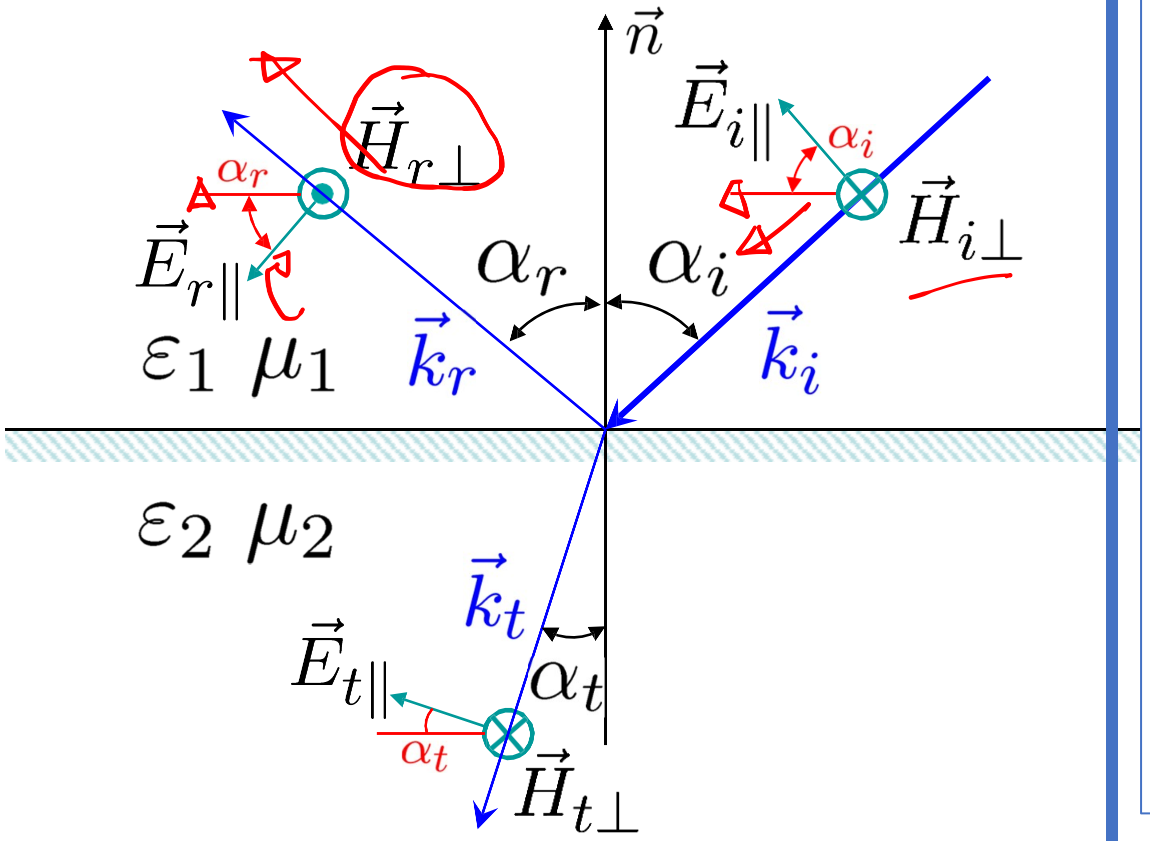
$$t^2 + r^2 = 1$$ **power balance**

Reflection and Refraction



Reflection and Refraction

Parallel Polarization



*** The magnetic field is perpendicular to the plane of incidence and the electric field is parallel to the plane of incidence.

$$\vec{E}_{1,tan} = \vec{E}_{2,tan}$$
$$\vec{H}_{1,tan} = \vec{H}_{2,tan}$$

According to Fig. 4.15 the boundary condition for the magnetic field reads

$$H_{i0\perp} - H_{r0\perp} = H_{t0\perp}$$

or with the impedance Z_1 for medium 1 and Z_2 for medium 2

$$\frac{E_{i0||} - E_{r0||}}{Z_1} = \frac{E_{t0||}}{Z_2} \tag{4.103}$$

and the condition that the tangential electric field components have to be continuous yields

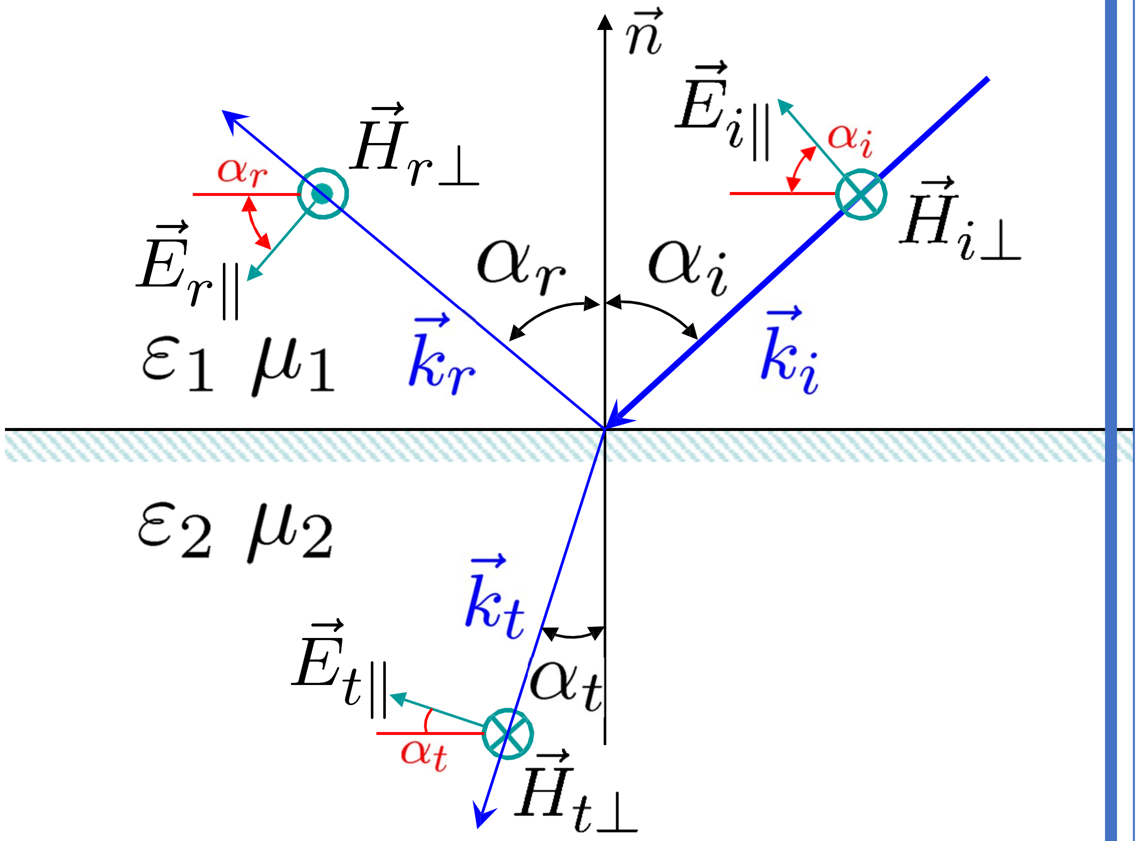
$$(E_{i0||} + E_{r0||}) \cos\alpha_1 = E_{t0||} \cos\alpha_2 \tag{4.104}$$

Replacing $E_{t0||}$ by combining these two conditions, we obtain the reflection coefficient for parallel polarization:

$$\boxed{r_{||} = \frac{E_{r0||}}{E_{i0||}} = \frac{Z_2\cos\alpha_2 - Z_1\cos\alpha_1}{Z_2\cos\alpha_2 + Z_1\cos\alpha_1}} \tag{4.105}$$

Reflection and Refraction

Parallel Polarization



*** The magnetic field is perpendicular to the plane of incidence and the electric field is parallel to the plane of incidence.

Solving the boundary conditions yields the reflection and transmission coefficients:

$$\boxed{r_{||} = \frac{E_{r0||}}{E_{i0||}} = \frac{Z_2 \cos\alpha_2 - Z_1 \cos\alpha_1}{Z_2 \cos\alpha_2 + Z_1 \cos\alpha_1}}$$

parallel polarization

$$\boxed{t_{||} = \frac{2\sqrt{Z_1 Z_2 \cos\alpha_1 \cos\alpha_2}}{Z_1 \cos\alpha_1 + Z_2 \cos\alpha_2}}$$

parallel polarization

$$t^2 + r^2 = 1$$

power balance

cos(A-B)-

Reflection and Refraction

4.6.3 Metallic, non-magnetic medium (medium no. 2):

cos

$$r_{||} = \frac{E_{r0||}}{E_{i0||}} = \frac{Z_2 \cos\alpha_2 - Z_1 \cos\alpha_1}{Z_2 \cos\alpha_2 + Z_1 \cos\alpha_1}$$

parallel polarization

$$t_{||} = \frac{2\sqrt{Z_1 Z_2}\cos\alpha_1\cos\alpha_2}{Z_1\cos\alpha_1 + Z_2\cos\alpha_2}$$

parallel polarization

sin

$$r_{\perp} = \frac{E_{r0\perp}}{E_{i0\perp}} = \frac{Z_2 \cos\alpha_1 - Z_1 \cos\alpha_2}{Z_2 \cos\alpha_1 + Z_1 \cos\alpha_2}$$

perpendicular polarization

$$t_{\perp} = \frac{2\sqrt{Z_1 Z_2}\cos\alpha_1\cos\alpha_2}{Z_2\cos\alpha_1 + Z_1\cos\alpha_2}$$

perpendicular polarization

$$Z_2 = \sqrt{\frac{\mu_0}{\underline{\varepsilon}}} \rightarrow 0$$

$$\underline{\varepsilon} = \varepsilon - j\frac{\sigma}{\omega} \approx -j\frac{\sigma}{\omega}$$

$$r_{\perp} = r_{||} = -1$$

$$t_{\perp} = t_{||} = 0$$

The incident wave is totally reflected.

μ_1, μ_0

μ_1, μ_0

Reflection and Refraction

4.6.4 Non-magnetic, dielectric media:

$z_1 \sin\alpha_1 = z_2 \sin\alpha_2$

$$\boxed{r_{||} = \frac{E_{r0||}}{E_{i0||}} = \frac{Z_2\cos\alpha_2 - Z_1\cos\alpha_1}{Z_2\cos\alpha_2 + Z_1\cos\alpha_1}}$$

parallel polarization

$$\boxed{t_{||} = \frac{2\sqrt{Z_1 Z_2}\cos\alpha_1\cos\alpha_2}{Z_1\cos\alpha_1 + Z_2\cos\alpha_2}}$$

parallel polarization

$$\boxed{r_{\perp} = \frac{E_{r0\perp}}{E_{i0\perp}} = \frac{Z_2\cos\alpha_1 - Z_1\cos\alpha_2}{Z_2\cos\alpha_1 + Z_1\cos\alpha_2}}$$

perpendicular polarization

$$\boxed{t_{\perp} = \frac{2\sqrt{Z_1 Z_2}\cos\alpha_1\cos\alpha_2}{Z_2\cos\alpha_1 + Z_1\cos\alpha_2}}$$

perpendicular polarization

$$\frac{Z_1}{Z_2} = \frac{\sqrt{\mu_0/\varepsilon_1}}{\sqrt{\mu_0/\varepsilon_2}} = \sqrt{\frac{\varepsilon_2}{\varepsilon_1}} = \frac{\sin\alpha_1}{\sin\alpha_2}$$

$Z_1 = (-\cdot)$

$$r_{\perp} = \frac{\sin\alpha_2\cos\alpha_1 - \sin\alpha_1\cos\alpha_2}{\sin\alpha_2\cos\alpha_1 + \sin\alpha_1\cos\alpha_2} = \frac{\sin(\alpha_2-\alpha_1)}{\sin(\alpha_2+\alpha_1)}$$

$$r_{||} = \frac{\sin\alpha_2\cos\alpha_2 - \sin\alpha_1\cos\alpha_1}{\sin\alpha_2\cos\alpha_2 + \sin\alpha_1\cos\alpha_1} = \frac{\tan(\alpha_2-\alpha_1)}{\tan(\alpha_2+\alpha_1)}$$

$$r_{\perp} = \frac{\sin(\alpha_2-\alpha_1)}{\sin(\alpha_2+\alpha_1)}$$

perpendicular polarization

$$r_{||} = \frac{\tan(\alpha_2-\alpha_1)}{\tan(\alpha_2+\alpha_1)}$$

parallel polarization

$$t^2 + r^2 = 1$$

Power balance!!

Reflection and Refraction

Perpendicularly impinging wave:

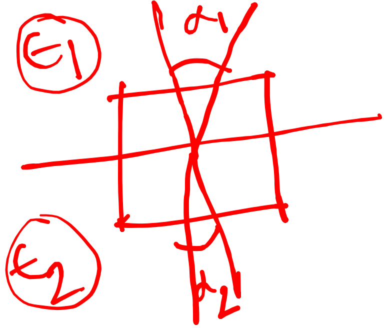
$$t_\perp = \frac{\sqrt{\sin^2(\alpha_2+\alpha_1)-\sin^2(\alpha_2-\alpha_1)}}{\sin(\alpha_2+\alpha_1)} \qquad (4.117)$$

$$t_{||} = \frac{\sqrt{\tan^2(\alpha_2+\alpha_1)-\tan^2(\alpha_2-\alpha_1)}}{\tan(\alpha_2+\alpha_1)} \qquad (4.118)$$

The special case of a **perpendicularly impinging wave** with $\alpha_1 = 0°$ results due to Snell's law in an angle of the transmitted wave of $\alpha_2 = \alpha_1 = 0°$. In this case eqns. (4.115) and (4.116) cannot be applied. Instead, we have to take the formulas in eqns. (4.100) and (4.105) and find with $\alpha_1 = \alpha_2 = 0°$ that both reflection coefficients are equal,

$$\boxed{r_\perp = \frac{E_{r0\perp}}{E_{i0\perp}} = \frac{Z_2\cos\alpha_1 - Z_1\cos\alpha_2}{Z_2\cos\alpha_1 + Z_1\cos\alpha_2}}$$

$$\boxed{t_\perp = \frac{2\sqrt{Z_1 Z_2\cos\alpha_1\cos\alpha_2}}{Z_2\cos\alpha_1 + Z_1\cos\alpha_2}}$$



ε1 ~ ε2

$$\alpha_2 = \alpha_1 = 0°$$

$$r_\perp = r_{||} = \frac{Z_2 - Z_1}{Z_2 + Z_1}$$

$$t_\perp = t_{||} = \frac{2\sqrt{Z_1 Z_2}}{Z_1 + Z_2}$$

- For any other impinging angle **α1 it cannot happen that α2 = α1** except for the trivial case ε1 = ε2 what is equivalent to the statement that both media are identical. Then no reflection occurs.

Reflection and Refraction

4.6.4 Non-magnetic, dielectric media (Brewster's Angle):

$$\frac{Z_1}{Z_2} = \frac{\sqrt{\mu_0/\varepsilon_1}}{\sqrt{\mu_0/\varepsilon_2}} = \sqrt{\frac{\varepsilon_2}{\varepsilon_1}} = \frac{\sin\alpha_1}{\sin\alpha_2}$$

$$r_\perp = \frac{\sin(\alpha_2 - \alpha_1)}{\sin(\alpha_2 + \alpha_1)}$$ **perpendicular polarization**

$$\boxed{r_{||} = \frac{\tan(\alpha_2 - \alpha_1)}{\tan(\alpha_2 + \alpha_1)}}$$ **parallel polarization** $= 0?$

$\tan 90^\circ = \infty$

$$t^2 + r^2 = 1$$ **Power balance!!**

$$\frac{Z_1}{Z_2} = \frac{\sqrt{\mu_0/\varepsilon_1}}{\sqrt{\mu_0/\varepsilon_2}} = \sqrt{\frac{\varepsilon_2}{\varepsilon_1}} = \frac{\sin\alpha_1}{\sin\alpha_2}$$

-1

One can also see that **for parallel polarization the reflection coefficient becomes zero if** $\alpha_1 + \alpha_2 = \frac{\pi}{2}$ (because $\tan\left(\frac{\pi}{2}\right) \to \infty$):

$$r_{||} = \frac{\tan(\alpha_2 - \alpha_1)}{\tan(\alpha_2 + \alpha_1)} \quad (4.121)$$

$$= \frac{\tan(2\alpha_2 - \frac{\pi}{2})}{\tan(\frac{\pi}{2})} \quad \text{for} \quad \alpha_1 + \alpha_2 = \frac{\pi}{2} \quad (4.122)$$

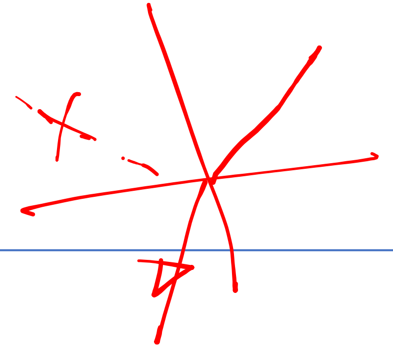
$$= 0 \quad \text{for} \quad \alpha_1 + \alpha_2 = \frac{\pi}{2} \quad (4.123)$$

With eq. (4.114) we get

$$\frac{\sin\alpha_1}{\sin\alpha_2} = \sqrt{\frac{\varepsilon_2}{\varepsilon_1}} = \frac{\sin\alpha_1}{\sin(\pi/2 - \alpha_1)} = \frac{\sin\alpha_1}{\cos\alpha_1} = \tan\alpha_1 \quad (4.124)$$

The angle α_1, defined by

$$\boxed{\tan\alpha_1 = \sqrt{\frac{\varepsilon_2}{\varepsilon_1}}} \quad (4.125)$$



is the so-called **Brewster's angle**.

Polarized angle

Parallel Polarization with NO Reflection if:

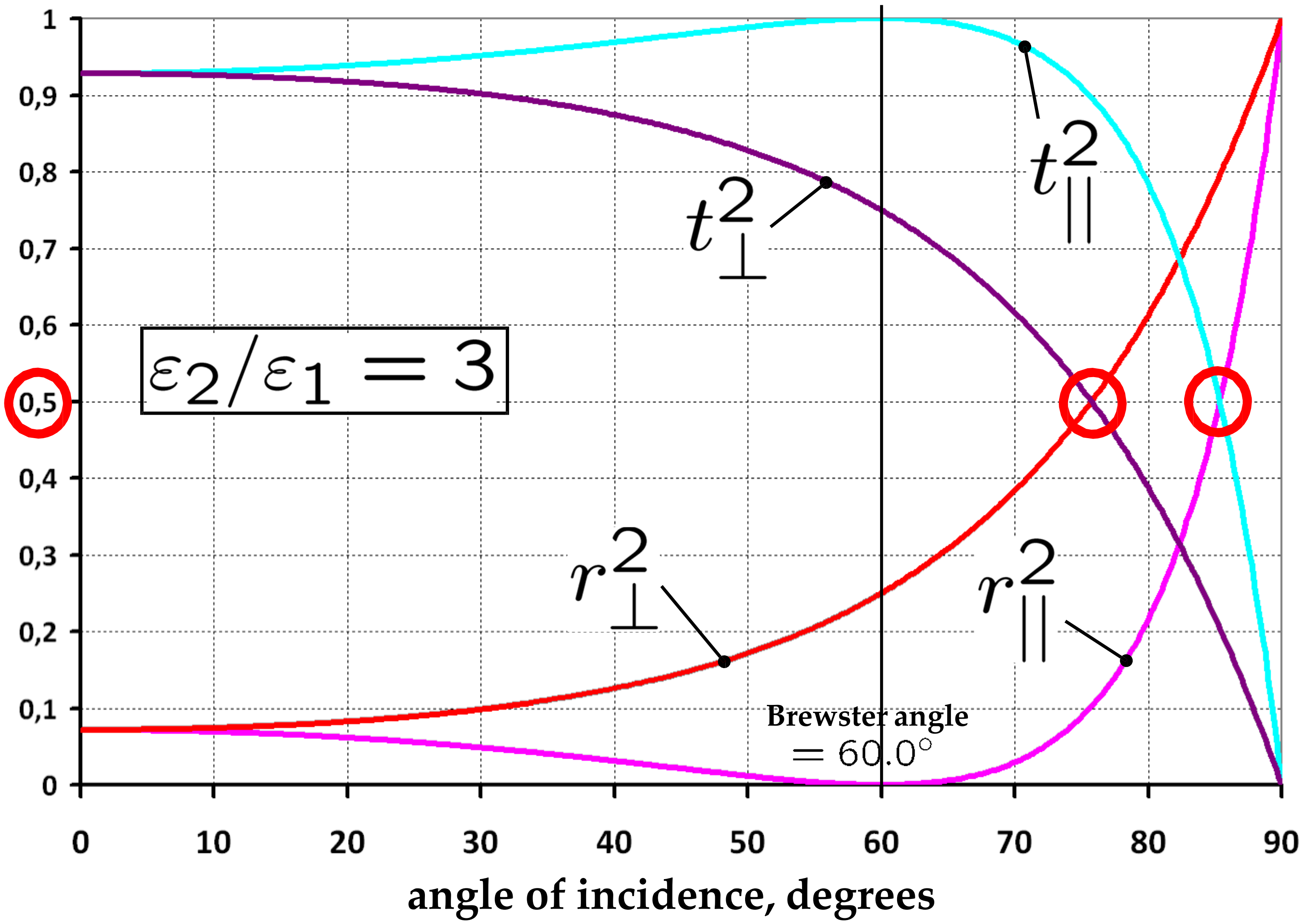
$$\boxed{\tan\alpha_1 = \sqrt{\frac{\varepsilon_2}{\varepsilon_1}}}$$

Brewster Angle

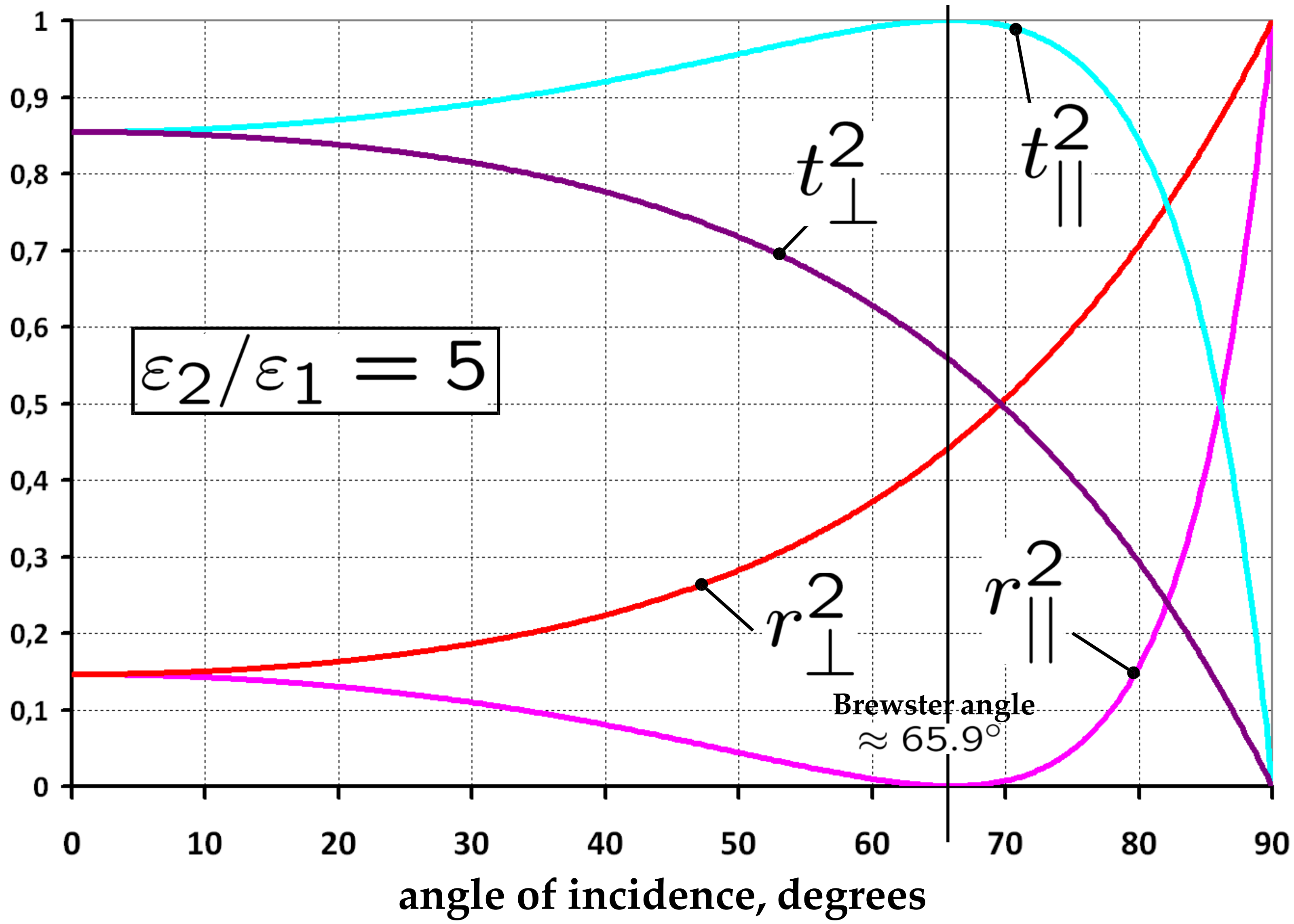
$$r_{\parallel} = 0,\ t_{\parallel} = 1$$

- **No reflection occurs** if the (parallel polarized) wave has an **angle of incidence that equals Brewster's angle.** In equivalent, Incident waves with **parallel polarization** (parallel w.r.t. the plane of incidence) are **totally transmitted** at a dielectric interface if the angle of incidence equals Brewster's angle.
- Brewster's angle is also called the **polarization angle.** If an electromagnetic wave (for example light) of unknown polarization impinges at Brewster's angle, the reflected wave can only be perpendicularly polarized because the parallel polarized portion of the wave will totally be transmitted due to the Brewster effect.

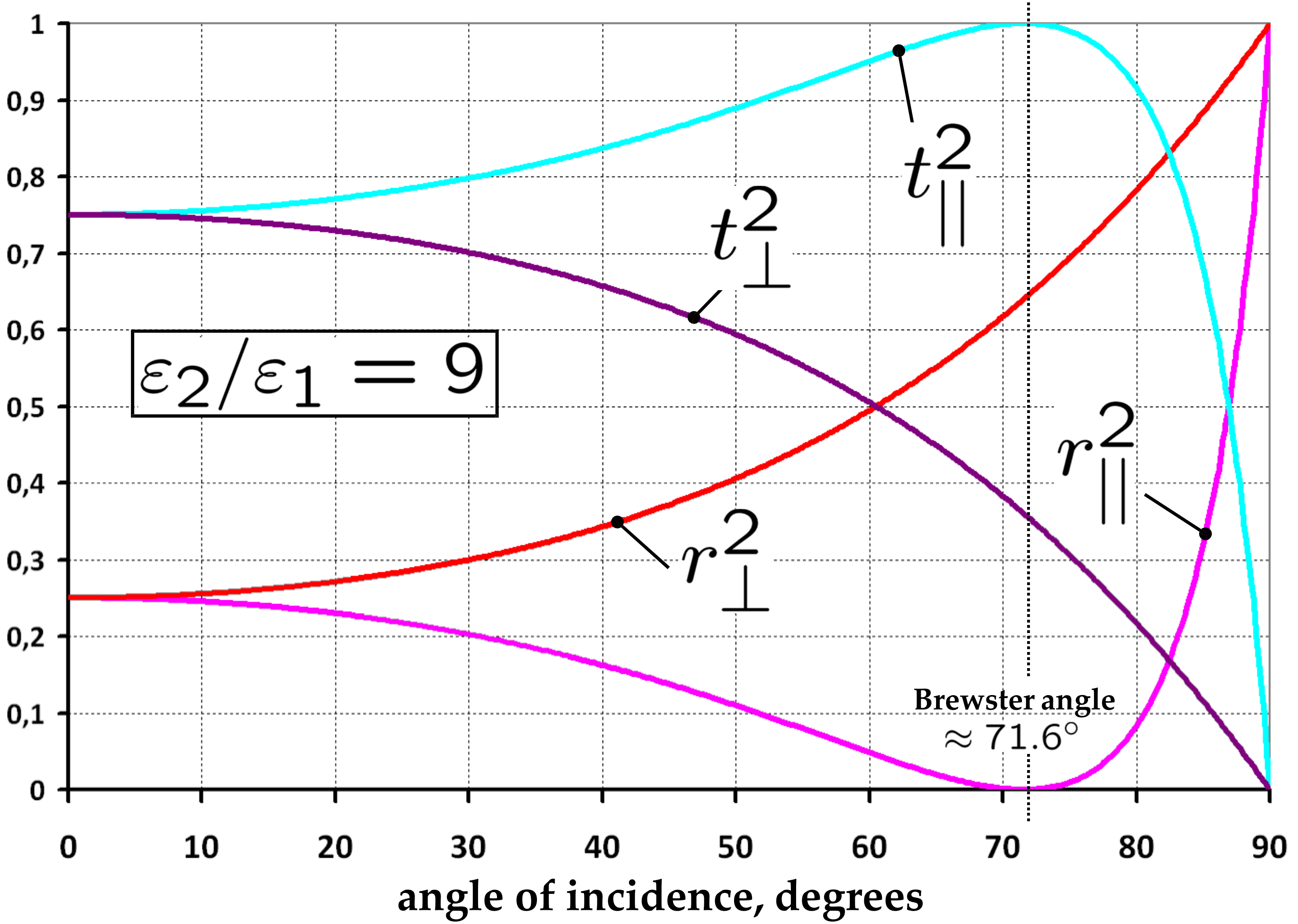
Reflection and Refraction



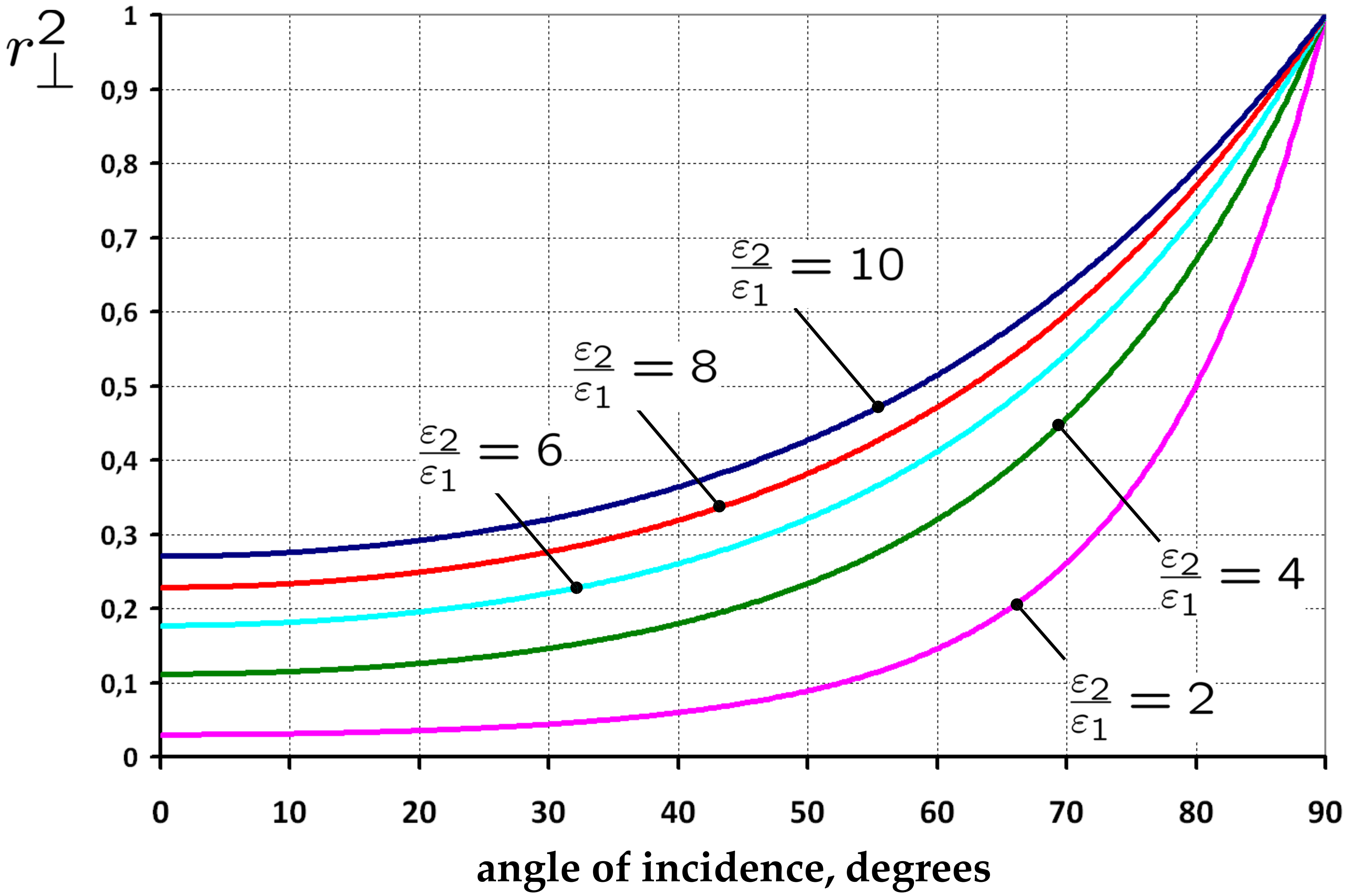
Reflection and Refraction



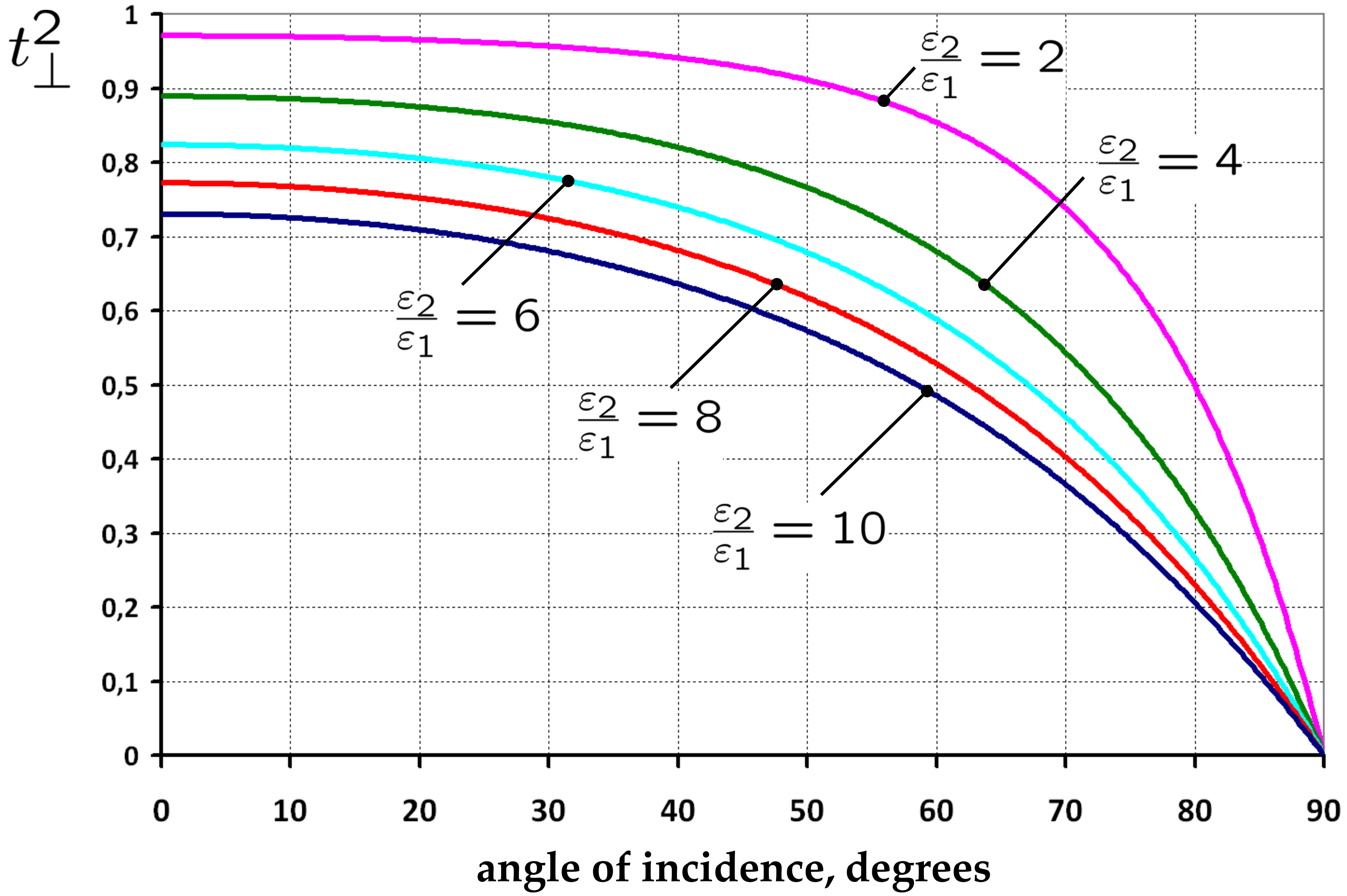
Reflection and Refraction



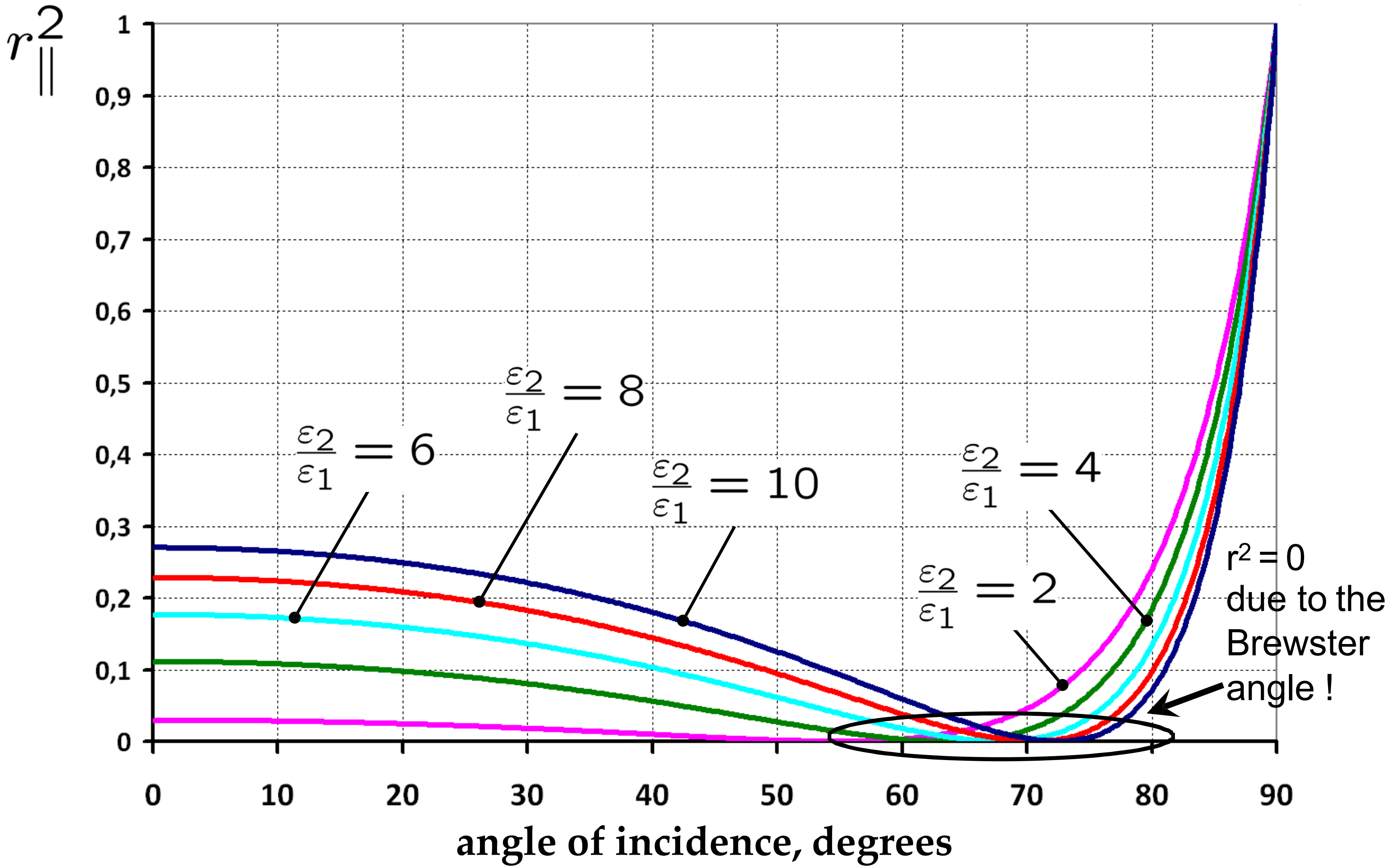
Reflection and Refraction



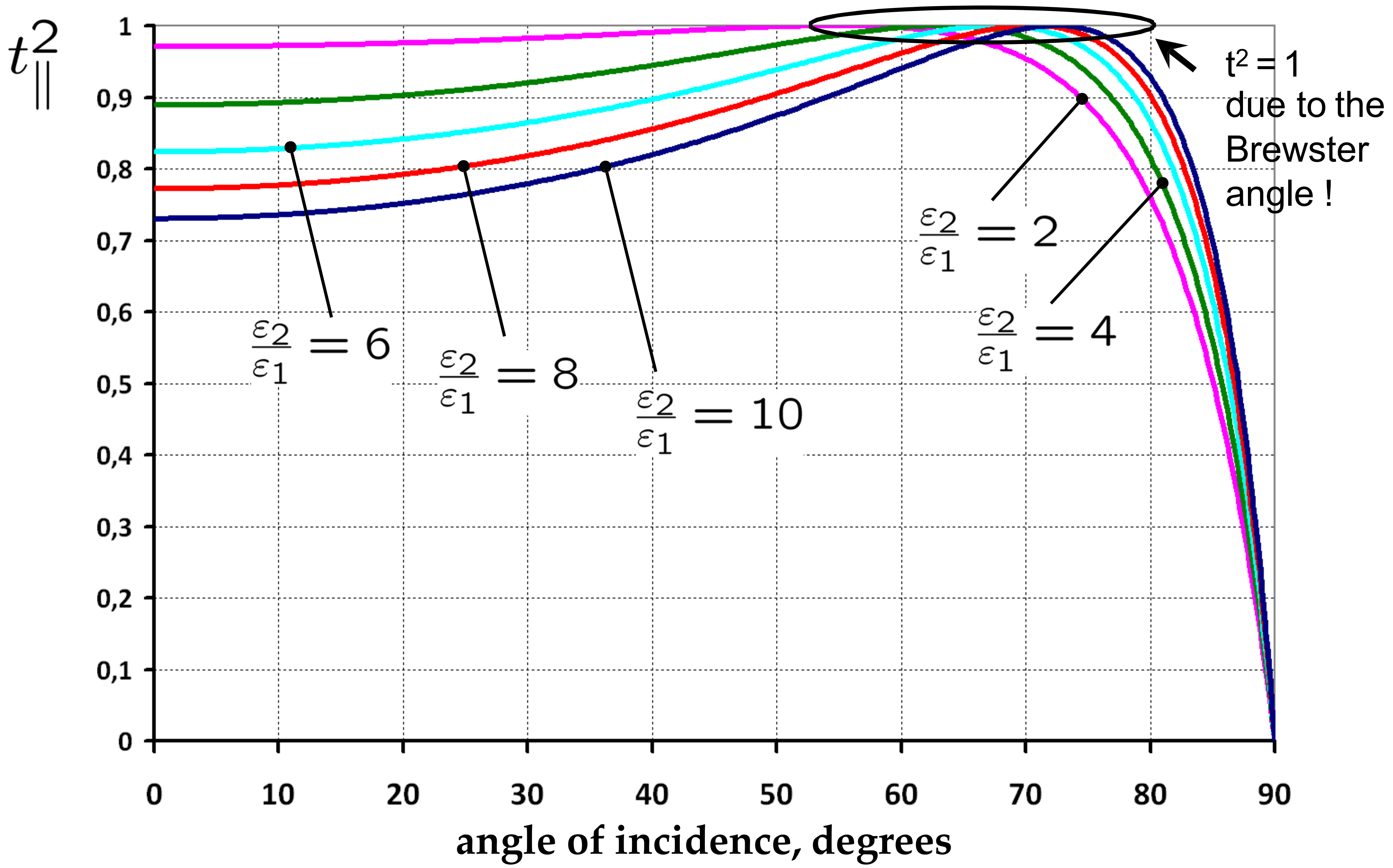
Reflection and Refraction



Reflection and Refraction



Reflection and Refraction



Reflection and Refraction

Effect of Brewster's angle in optics:

Camera without a polarizer.

Camera with a polarizer. There is no glass. (means no reflection has been found)

4.5 Problem 5

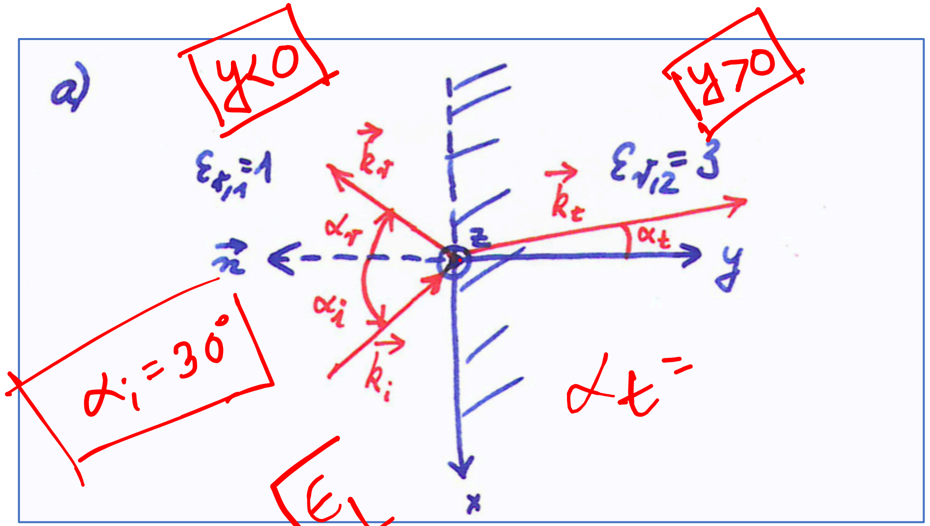
A dielectric medium with a relative permittivity of $\varepsilon_r = 3$ fills the half space $y \geq 0$. A plane TEM wave impinges from free space onto the boundary.

a) The wave vector is in the $x - y$ plane and the angle incidence (with respect to the normal vector) is $30°$. Sketch the wave vectors of the incident, reflected and refracted waves.

b) Calculate the angles of reflection and refraction for angles of incidence of $30°$ and $60°$, respectively. In which planes are the reflected and refracted wave vectors?

c) How much power from an incident, linearly polarized wave is transmitted into the dielectric medium for an angle of incidence of $30°$ and tilt angle of $\phi = 45°$ (i.e., with respect to the orientation of the linear polarization)?

d) Under which conditions will the incident wave be totally transmitted into the dielectric material? Does this hold for all frequencies? Sketch the electric field vectors for this case.



$$\frac{\sin\alpha_t}{\sin\alpha_i} = \sqrt{\frac{\varepsilon_1}{\varepsilon_2}}$$

b)

$\alpha_i = 30°$

$\alpha_r = \alpha_i = 30°$

$$\frac{\sin\alpha_t}{\sin\alpha_i} = \sqrt{\frac{\varepsilon_{r,1}}{\varepsilon_{r,2}}} = \sqrt{\frac{1}{3}}$$

$$\sin\alpha_t = \frac{1}{\sqrt{3}}\sin\alpha_i = \frac{1}{\sqrt{3}}\sin 30° = \frac{1}{2\sqrt{3}}$$

$$\Rightarrow \alpha_t = 16.78°$$

$\alpha_i = 60°$

$\alpha_r = \alpha_i = 60°$

$$\sin\alpha_t = \frac{1}{\sqrt{3}}\sin\alpha_i = \frac{1}{\sqrt{3}}\sin 60° = \frac{1}{\sqrt{3}}\cdot\frac{1}{2}\sqrt{3} = \frac{1}{2}$$

$$\Rightarrow \alpha_t = 30°$$

The reflected and the refracted wave vectors are in the same plane as the incident wave vector, i.e., the x-y-plane.

4.5 Problem 5

A dielectric medium with a relative permittivity of $\varepsilon_r = 3$ fills the half space $y \geq 0$. A plane TEM wave impinges from free space onto the boundary.

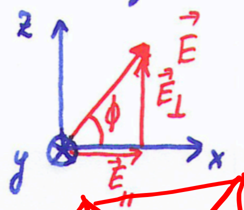
a) The wave vector is in the $x - y$ plane and the angle incidence (with respect to the normal vector) is $30°$. Sketch the wave vectors of the incident, reflected and refracted waves.

b) Calculate the angles of reflection and refraction for angles of incidence of $30°$ and $60°$, respectively. In which planes are the reflected and refracted wave vectors?

c) How much power from an incident, linearly polarized wave is transmitted into the dielectric medium for an angle of incidence of $30°$ and tilt angle of $\phi = 45°$ (i.e., with respect to the orientation of the linear polarization)?

d) Under which conditions will the incident wave be totally transmitted into the dielectric material? Does this hold for all frequencies? Sketch the electric field vectors for this case.

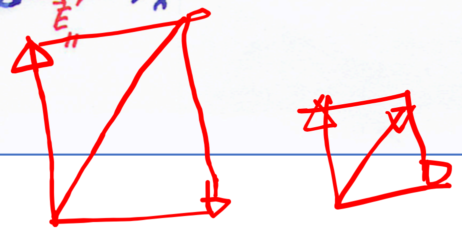
c) tilt angle $\phi = 45°$ (i.e., angle of polarization)



$\vec{E} = \vec{E}_\parallel + \vec{E}_\perp \qquad \sphericalangle(\vec{E}_\perp, \vec{E}_\parallel) = 45°$

$\Rightarrow |\vec{E}_\parallel| = |\vec{E}_\perp|$

$\Rightarrow$ Both portions contain the same amount of power.



$\alpha_1 = \alpha_i = 30°$; $\alpha_2 = \alpha_t = 16.78°$

transmission coefficients:

perpendicular portion: $t_\perp = \dfrac{2\sqrt{Z_1 Z_2 \cos\alpha_1 \cos\alpha_2}}{Z_2 \cos\alpha_1 + Z_1 \cos\alpha_2}$

$Z_1 = \sqrt{\dfrac{\mu_0}{\varepsilon_0}} \approx 377\,\Omega$; $Z_2 = \sqrt{\dfrac{\mu_0}{\varepsilon_0 \varepsilon_{r2}}} = \dfrac{Z_1}{\sqrt{3}} \approx 217{,}66\,\Omega$

$\Rightarrow t_\perp \approx 0.949$

parallel ~~pts~~ portion: $t_\parallel = \dfrac{2\sqrt{Z_1 Z_2 \cos\alpha_1 \cos\alpha_2}}{Z_1 \cos\alpha_1 + Z_2 \cos\alpha_2}$

$\Rightarrow t_\parallel = 0.975$

transmitted powers:

$t_\perp^2 = 0.901$; $t_\parallel^2 = 0.951$

total transmitted power:

$t_{tot}^2 = \dfrac{t_\parallel^2 + t_\perp^2}{2} = 0.926$

$\Rightarrow$ 92.6 % of the incident power is transmitted.

Note: the calculation of the total power with the individual powers (and not via the vector fields) is allowed because both portions, parallel and perpendicular, are in phase in both media and experience the same phase shifts from the refraction.

4.5 Problem 5

A dielectric medium with a relative permittivity of $\varepsilon_r = 3$ fills the half space $y \geq 0$. A plane TEM wave impinges from free space onto the boundary.

a) The wave vector is in the $x-y$ plane and the angle incidence (with respect to the normal vector) is $30°$. Sketch the wave vectors of the incident, reflected and refracted waves.

b) Calculate the angles of reflection and refraction for angles of incidence of $30°$ and $60°$, respectively. In which planes are the reflected and refracted wave vectors?

c) How much power from an incident, linearly polarized wave is transmitted into the dielectric medium for an angle of incidence of $30°$ and tilt angle of $\phi = 45°$ (i.e., with respect to the orientation of the linear polarization)?

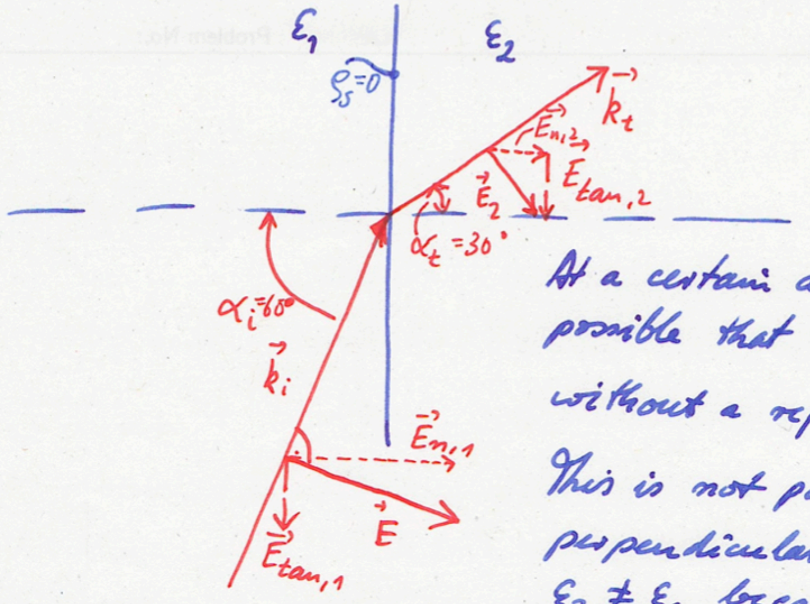
d) Under which conditions will the incident wave be totally transmitted into the dielectric material? Does this hold for all frequencies? Sketch the electric field vectors for this case.

d) For parallel polarization the wave is totally transmitted if the angle of incidence equals the Brewster angle,

$$\tan\alpha_i = \sqrt{\frac{\varepsilon_2}{\varepsilon_1}} = \sqrt{3} \; ; \Rightarrow \alpha_i = 60°$$

This holds for all frequencies (if ε_r is constant over frequency).

Why? → Boundary Condition $\vec{E}_{tan_1} = \vec{E}_{tan_2}$! 25



At a certain angle α_i it is possible that $\vec{E}_{tan,1} = \vec{E}_{tan,2}$ without a reflected wave.

This is not possible for perpendicular polarization if $\varepsilon_2 \neq \varepsilon_1$ because the total $\vec{E}$ vectors are tangential and they have to be different for different $\varepsilon_1, \varepsilon_2$.

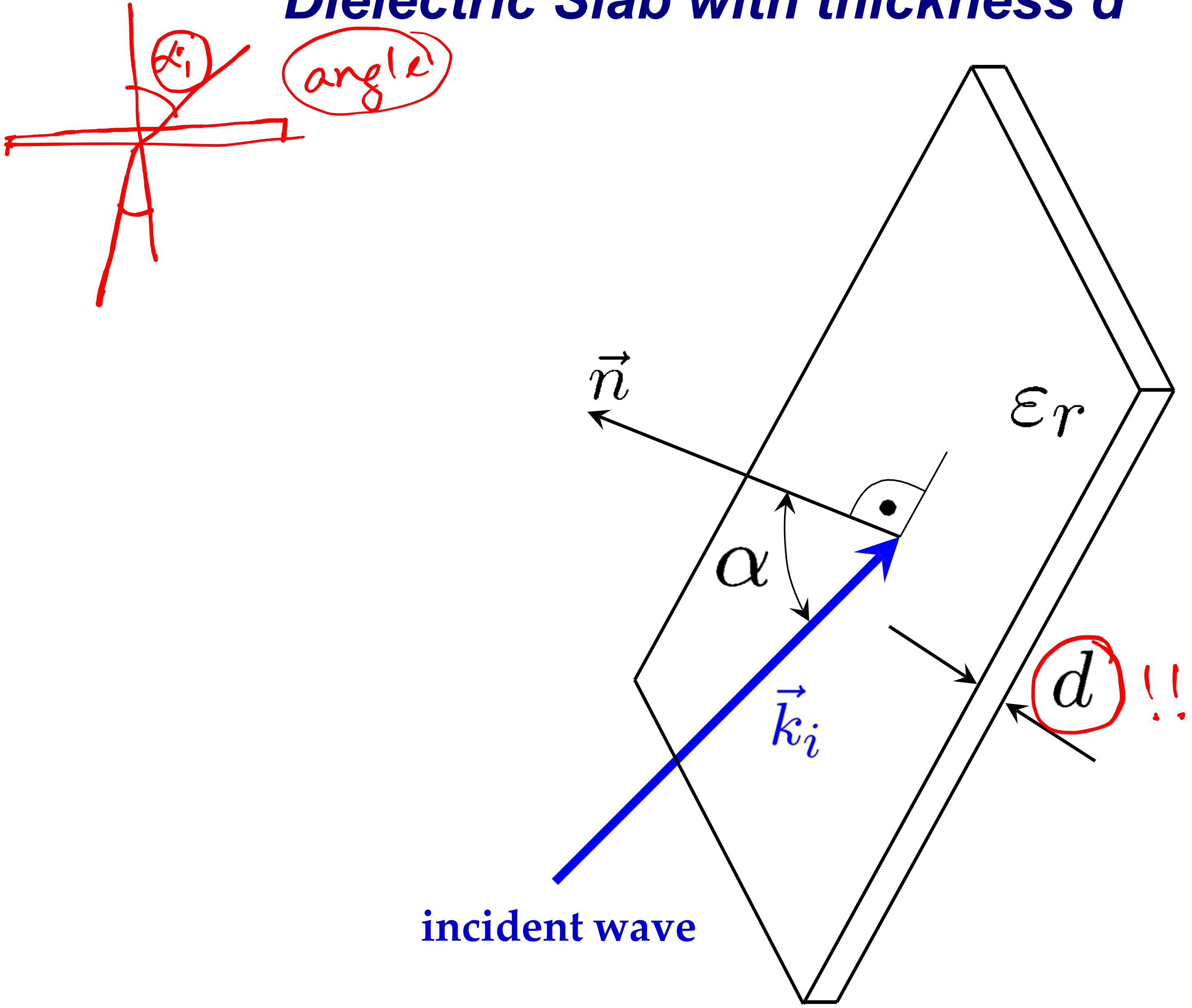
$\rho_s = 0$

$$\Rightarrow |\vec{D}_{n,1}| = |\vec{D}_{n,2}|$$

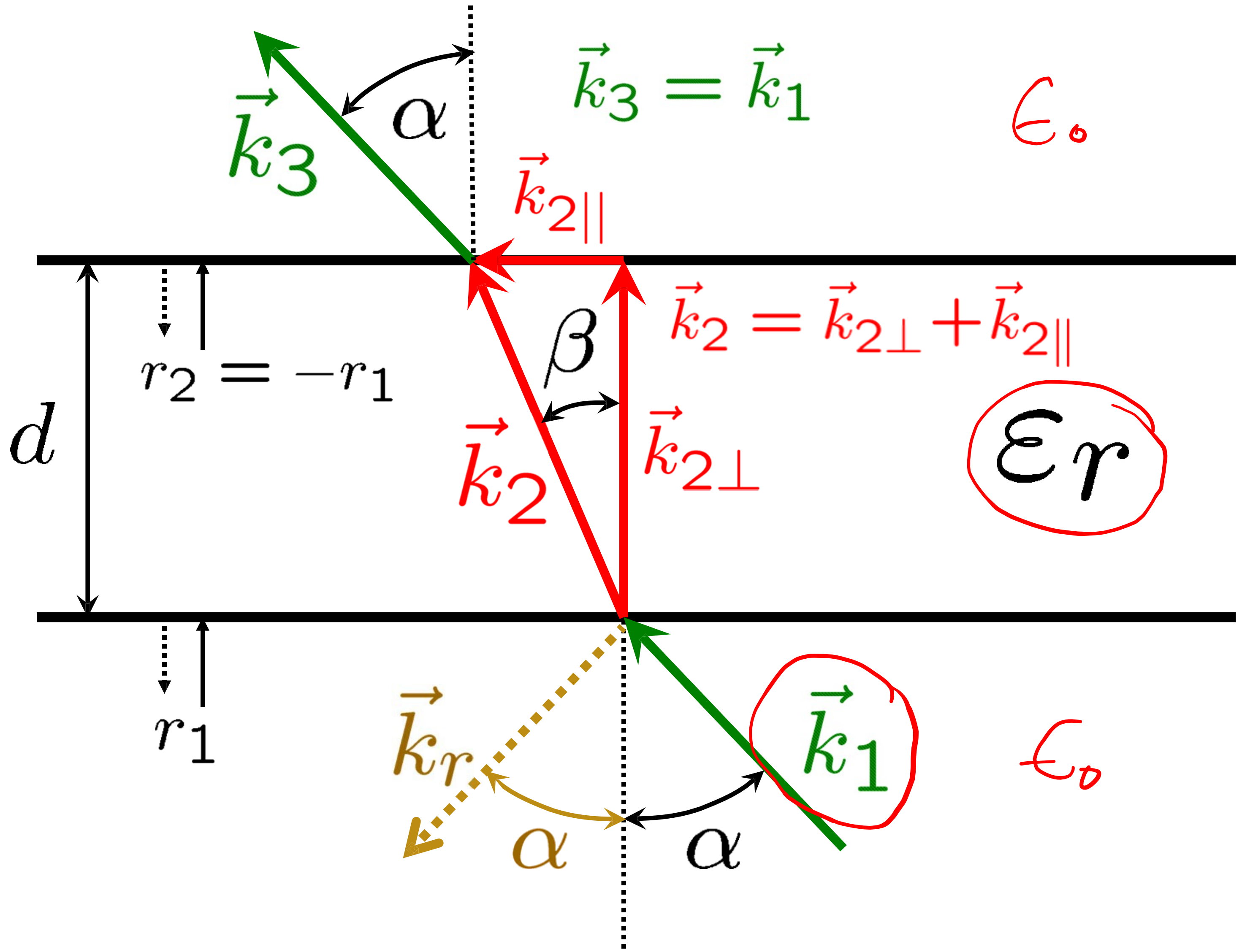
$$\varepsilon_1 |\vec{E}_{n,1}| = \varepsilon_2 |\vec{E}_{n,2}|$$

$$\Rightarrow |\vec{E}_{n,2}| = \frac{\varepsilon_1}{\varepsilon_2} |\vec{E}_{n,1}| = \frac{1}{3} |\vec{E}_{n,1}|$$

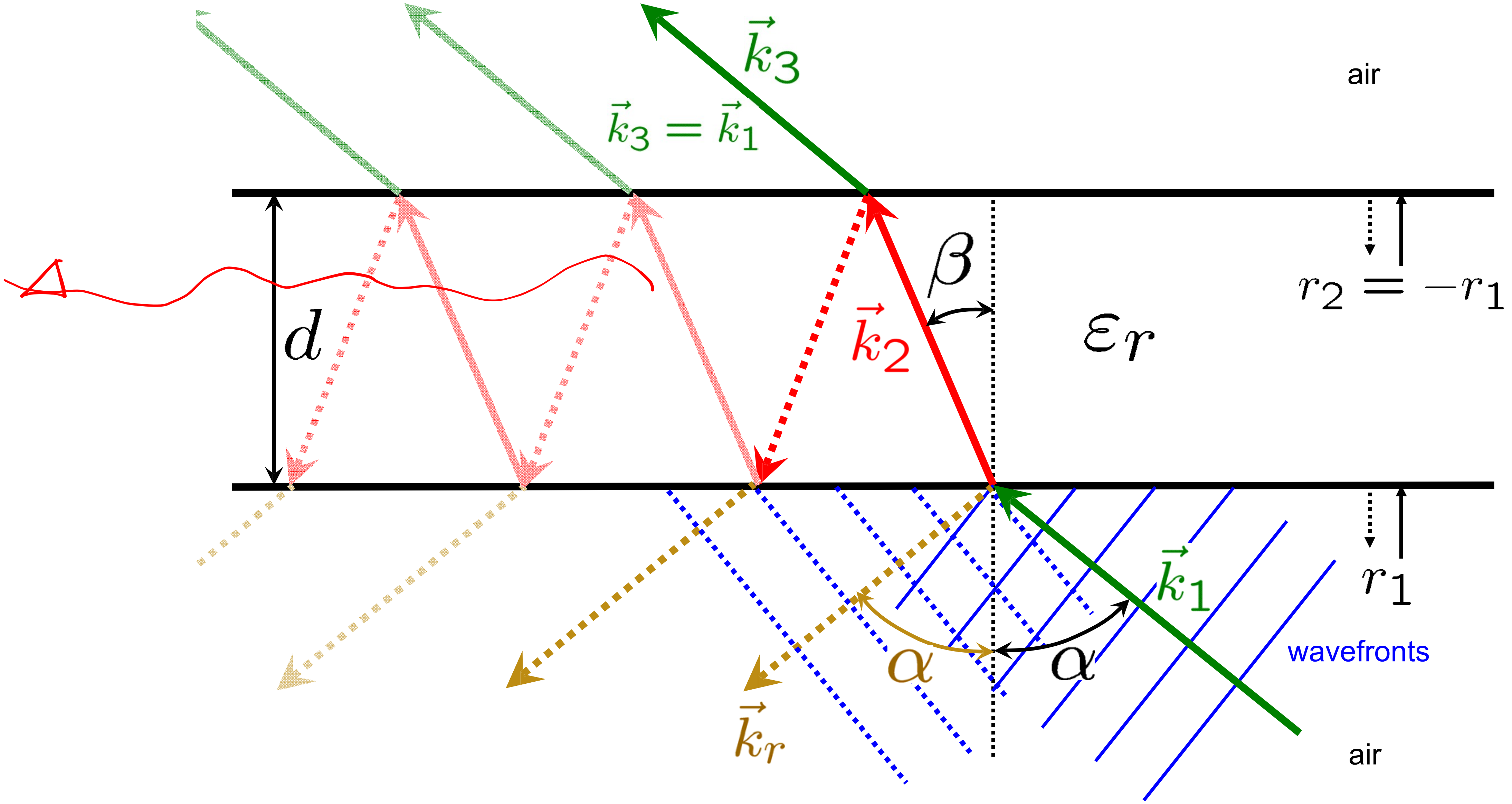
Dielectric Slab with thickness d



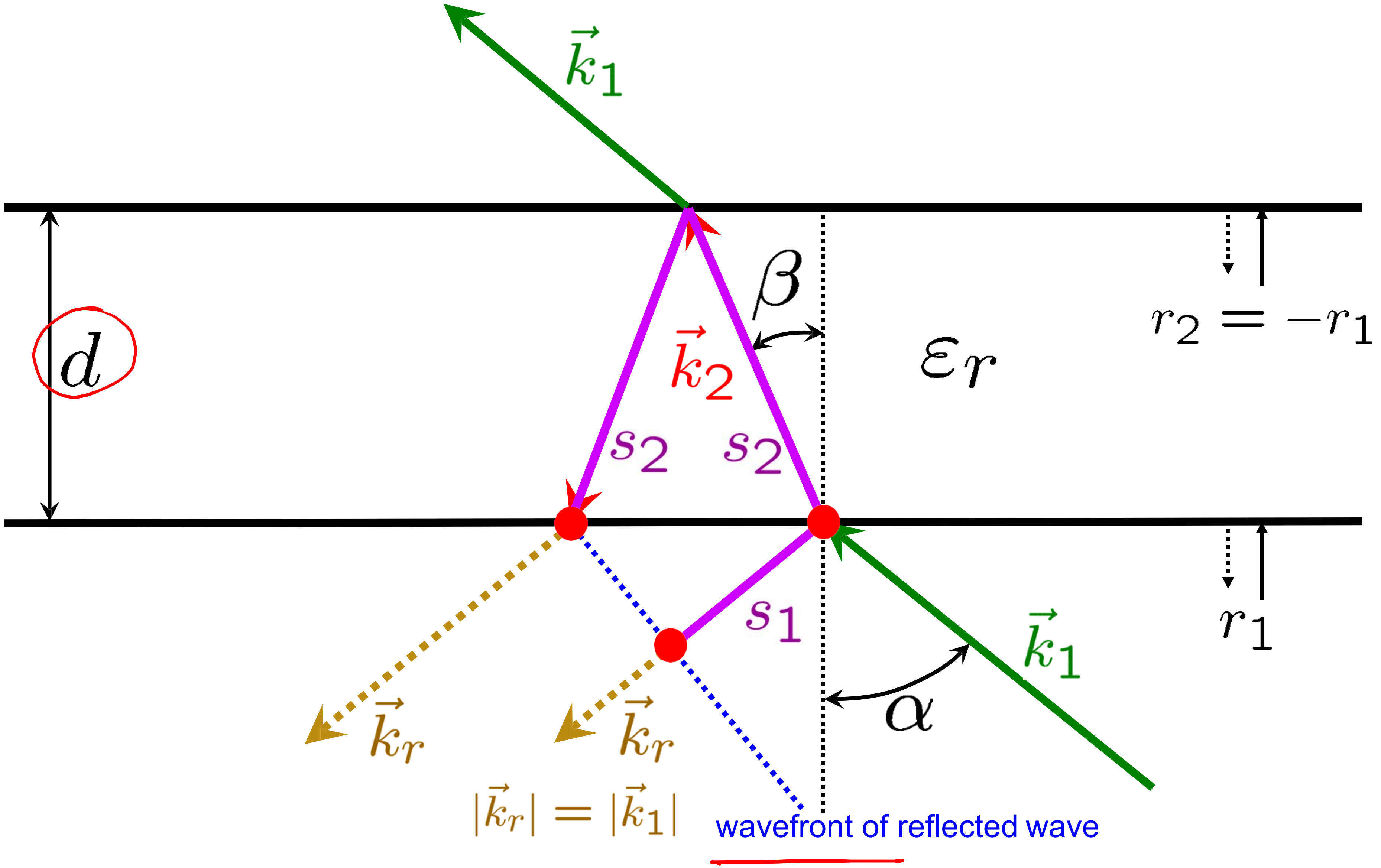
Dielectric Slab



Dielectric Slab



Dielectric Slab



Dielectric Slab

d↑ greatest?

Total reflection of a dielectric slab (e.g., a radome) with thickness d can be avoided by:

$$d = m \cdot \frac{\lambda_0}{2} \cdot \frac{1}{\sqrt{\varepsilon_r - \sin^2\alpha}}, \quad m = 1, 2, \ldots$$

$d = m\frac{\lambda_0}{2} \times \frac{1}{\sqrt{\varepsilon_r}}$

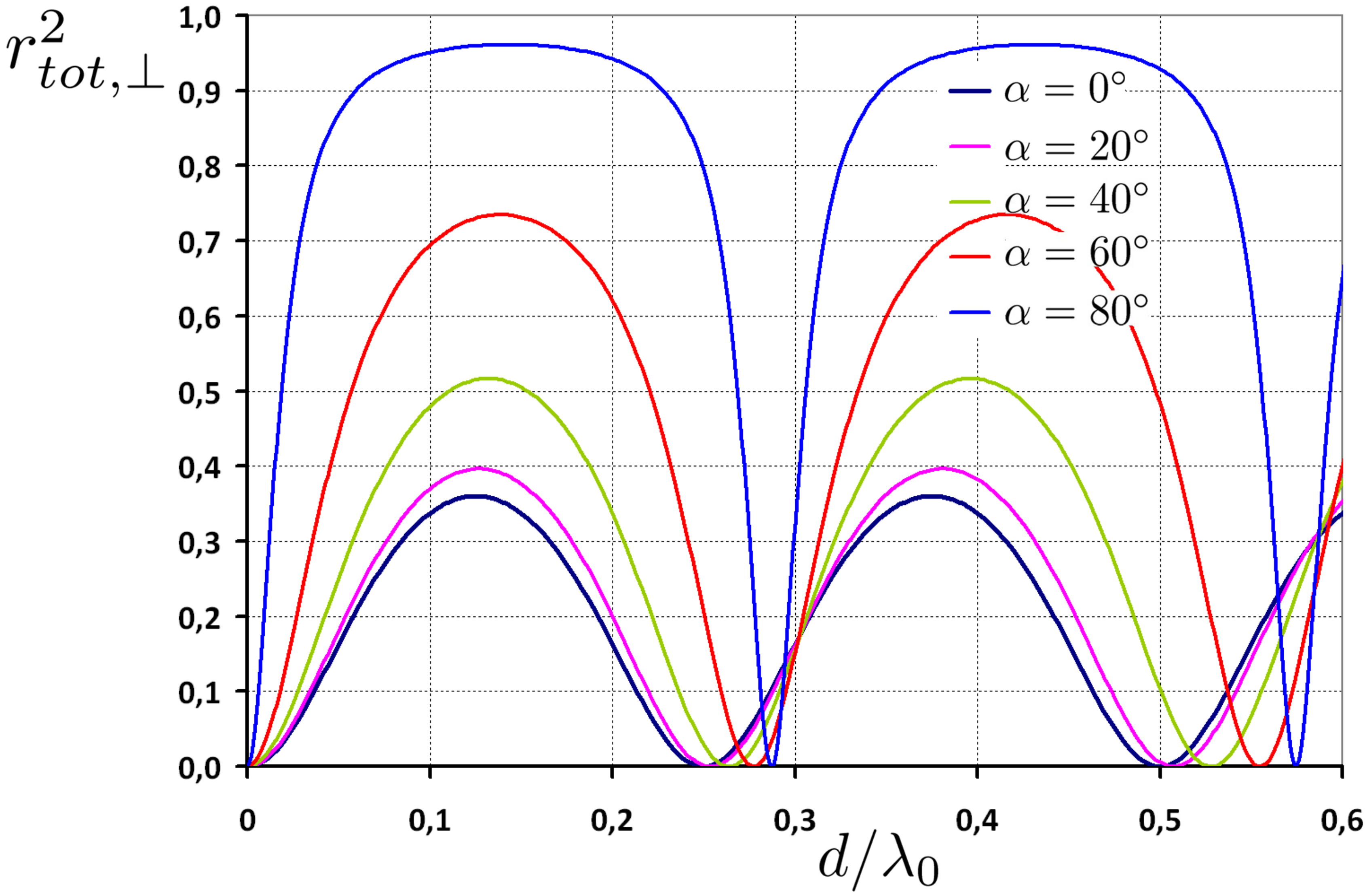
0

slab এর

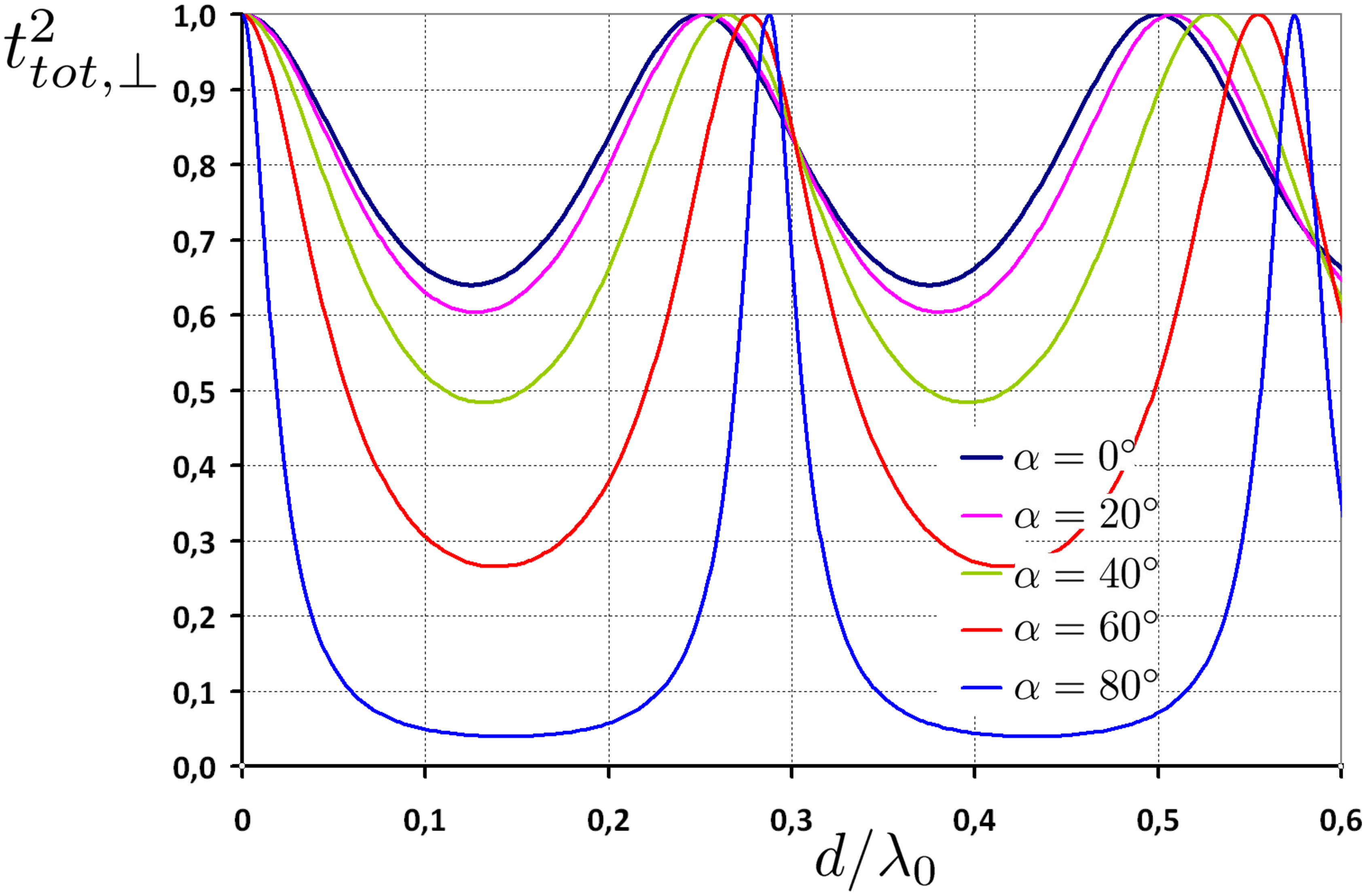
If this condition is fulfilled the power is totally transmitted through the (lossless) dielectric slab.

*** We have seen that at the case of Brewer's Angle!

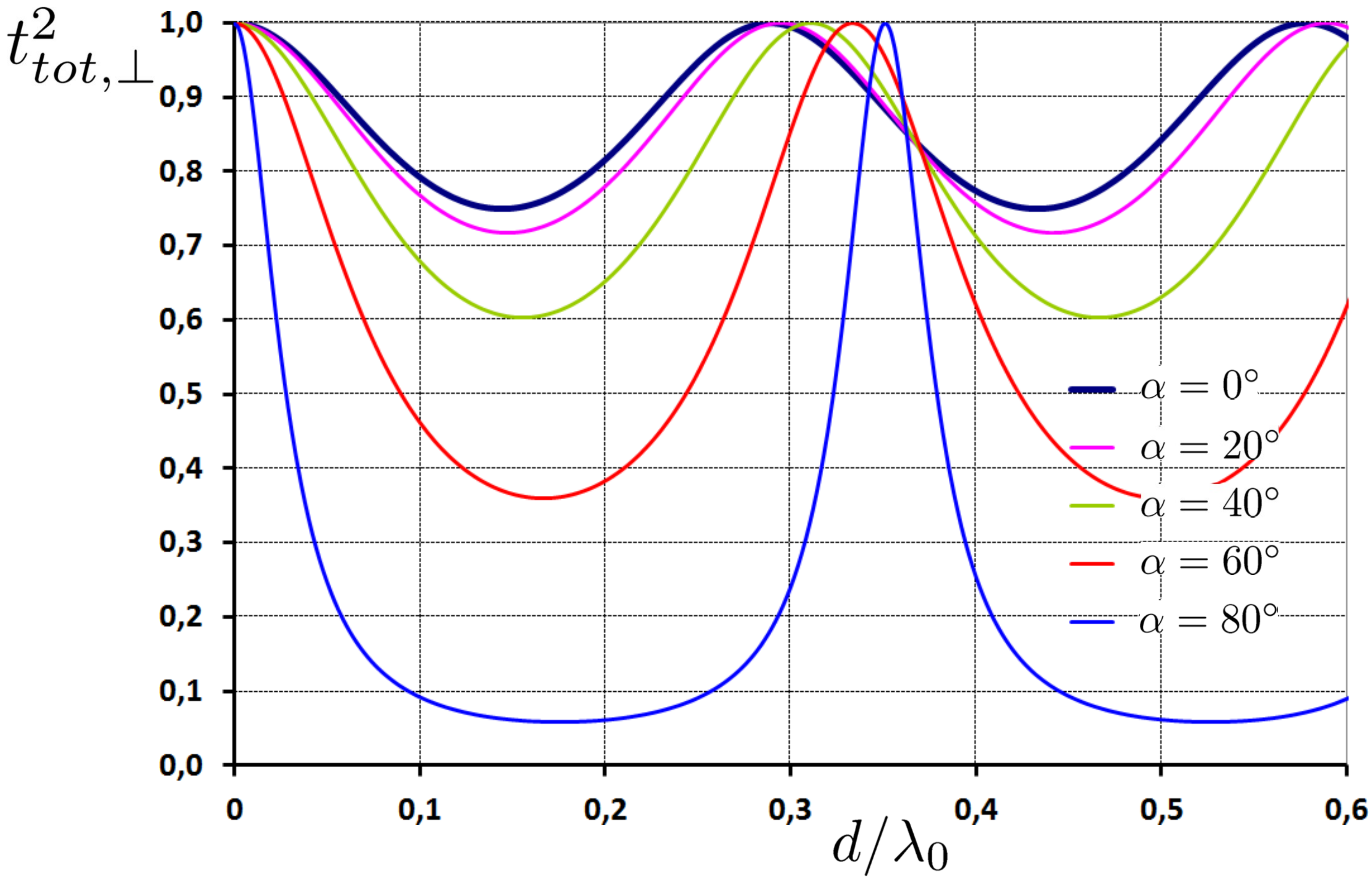
Lossless Dielectric Slab with $\varepsilon_r = 4.0$



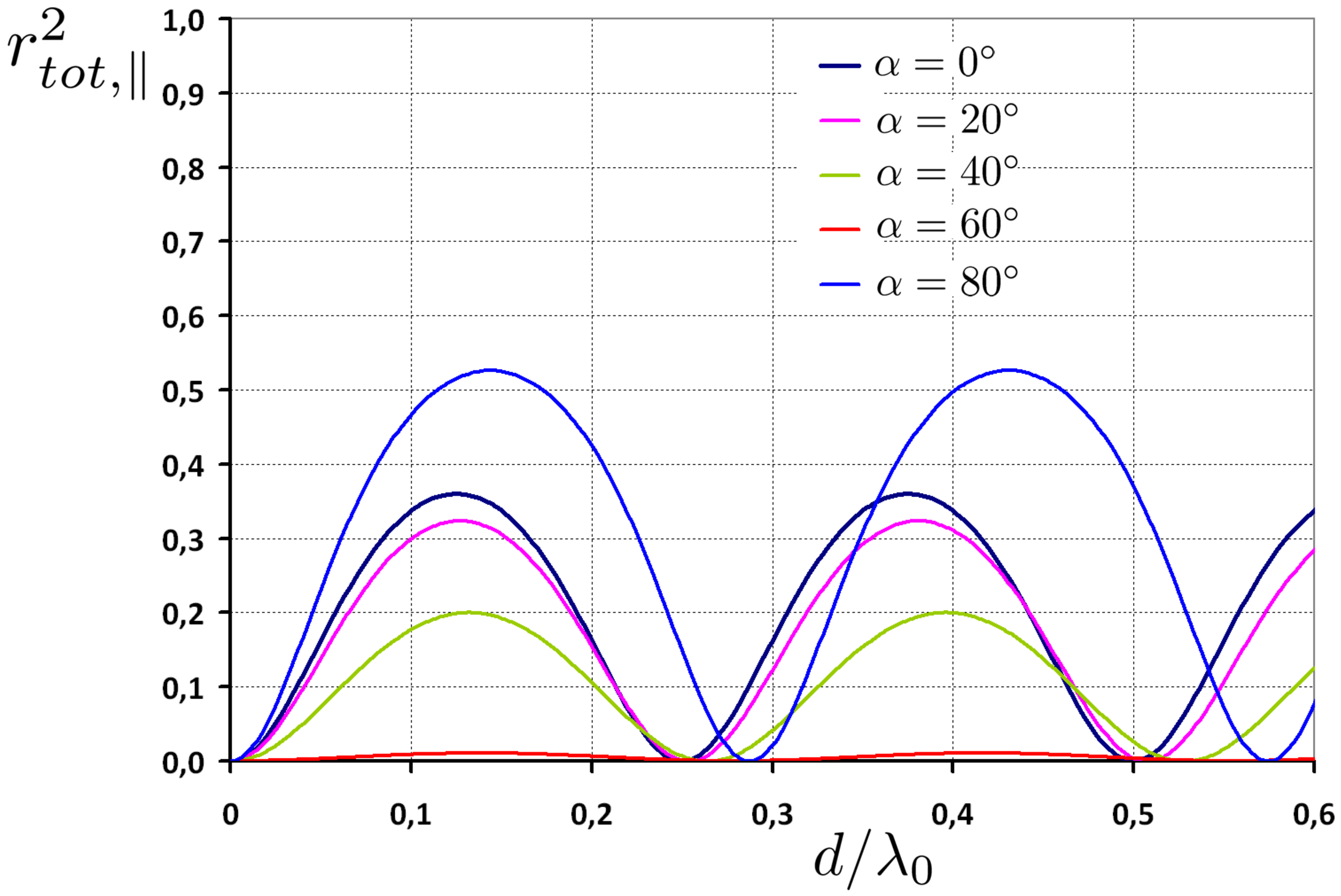
Lossless Dielectric Slab with $\varepsilon_r = 4.0$



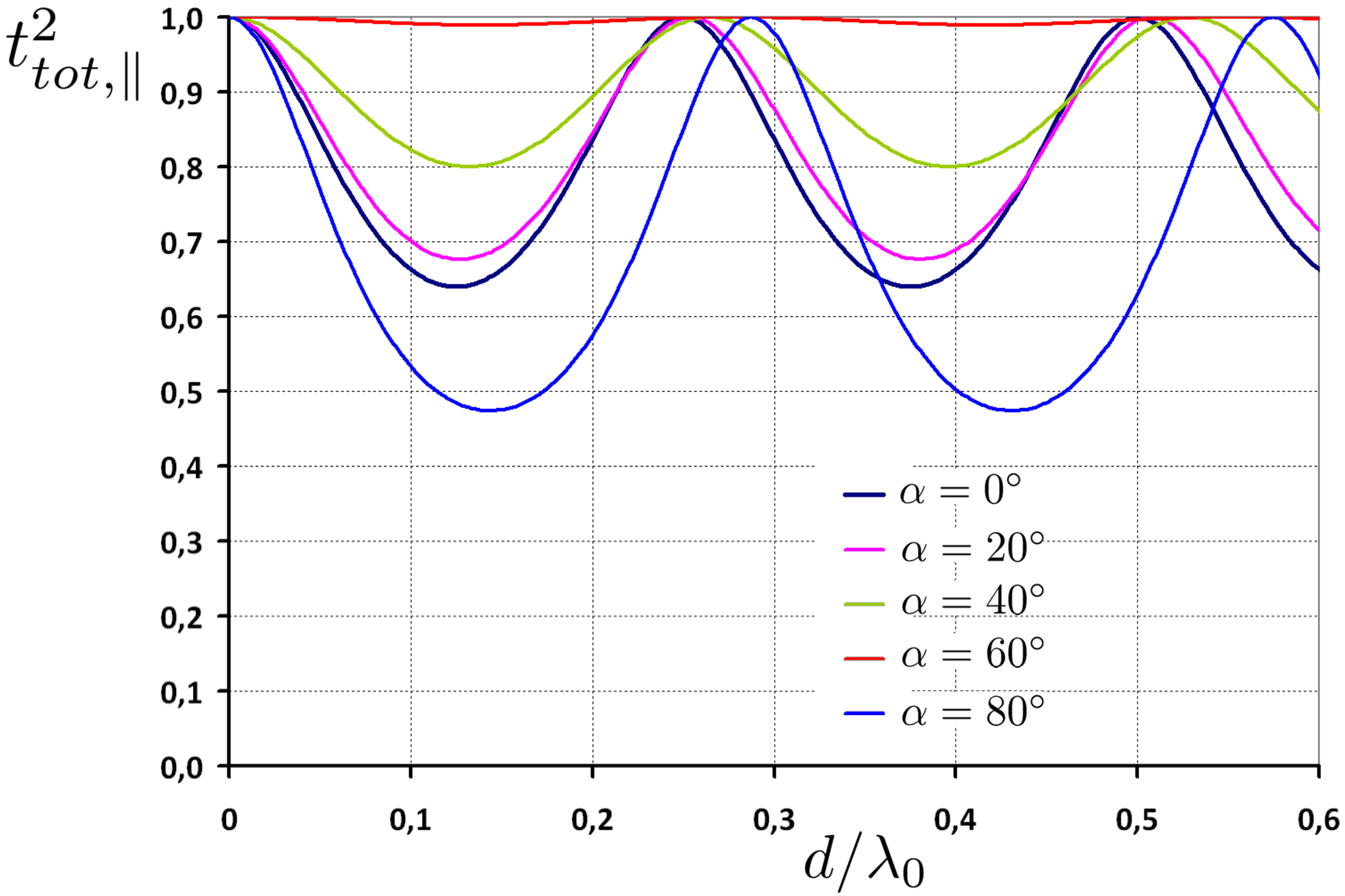
Lossless Dielectric Slab with $\varepsilon_r = 3.0$



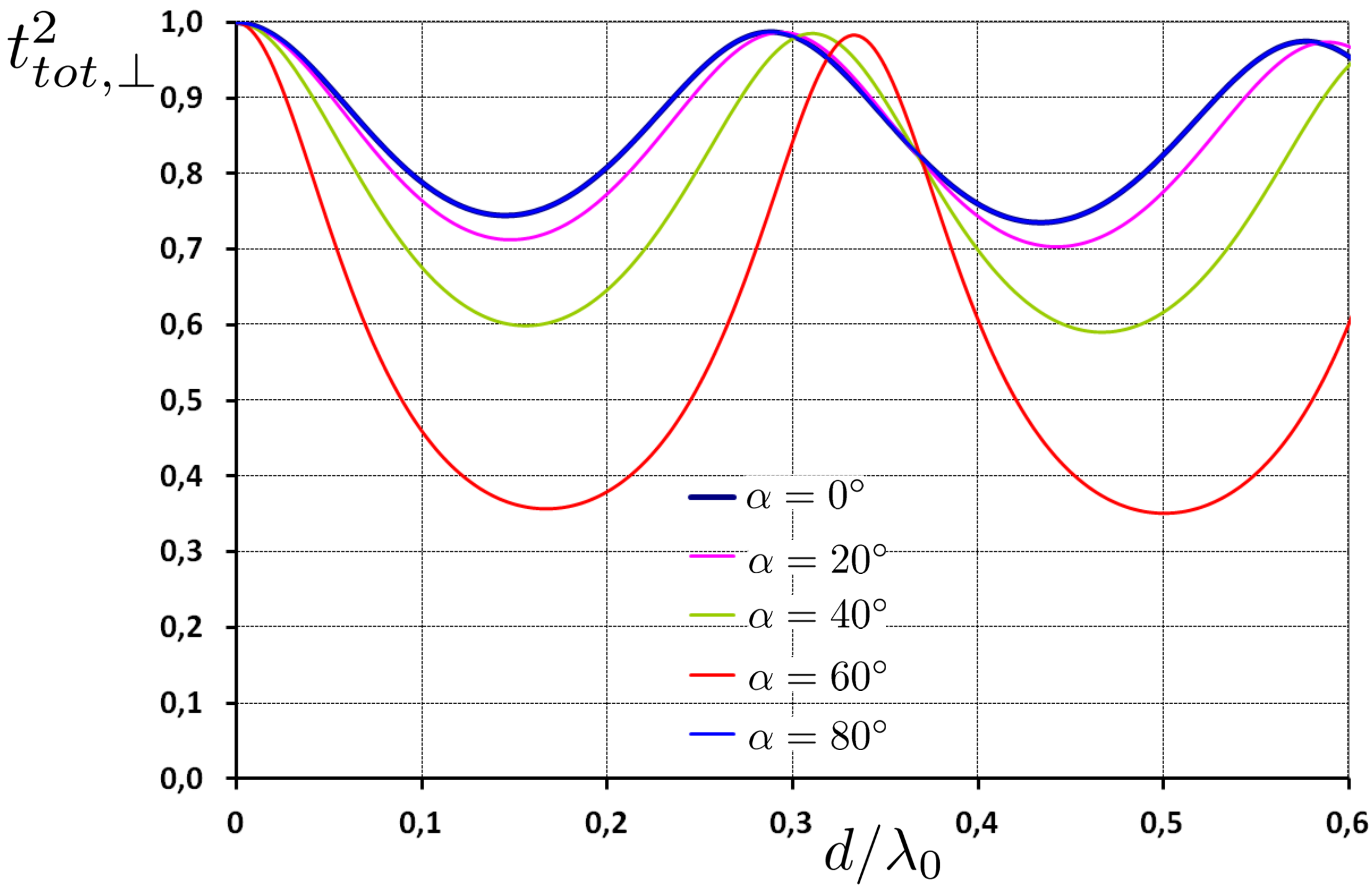
Lossless Dielectric Slab with $\varepsilon_r = 4.0$



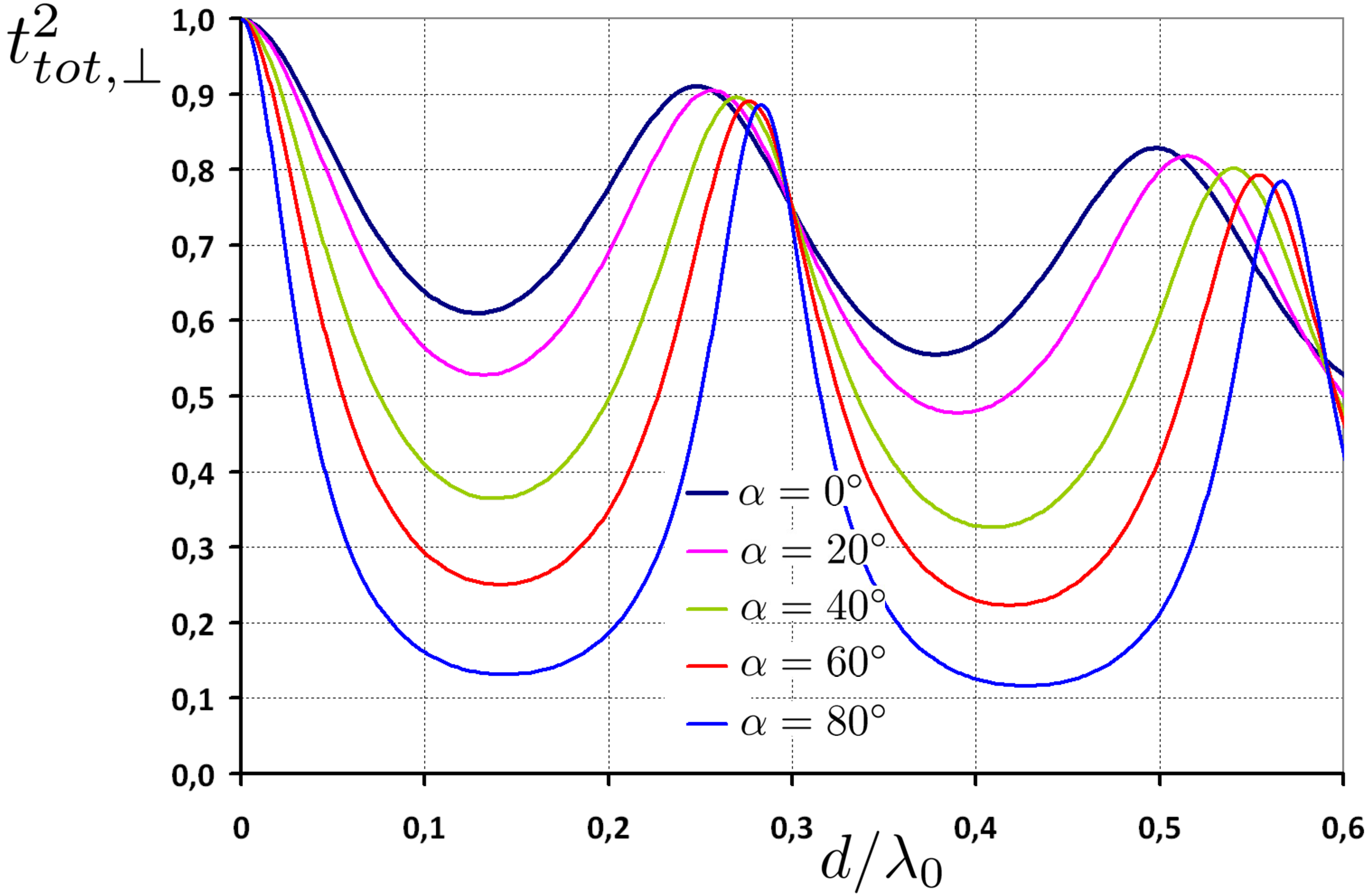
Lossless Dielectric Slab with $\varepsilon_r = 4.0$



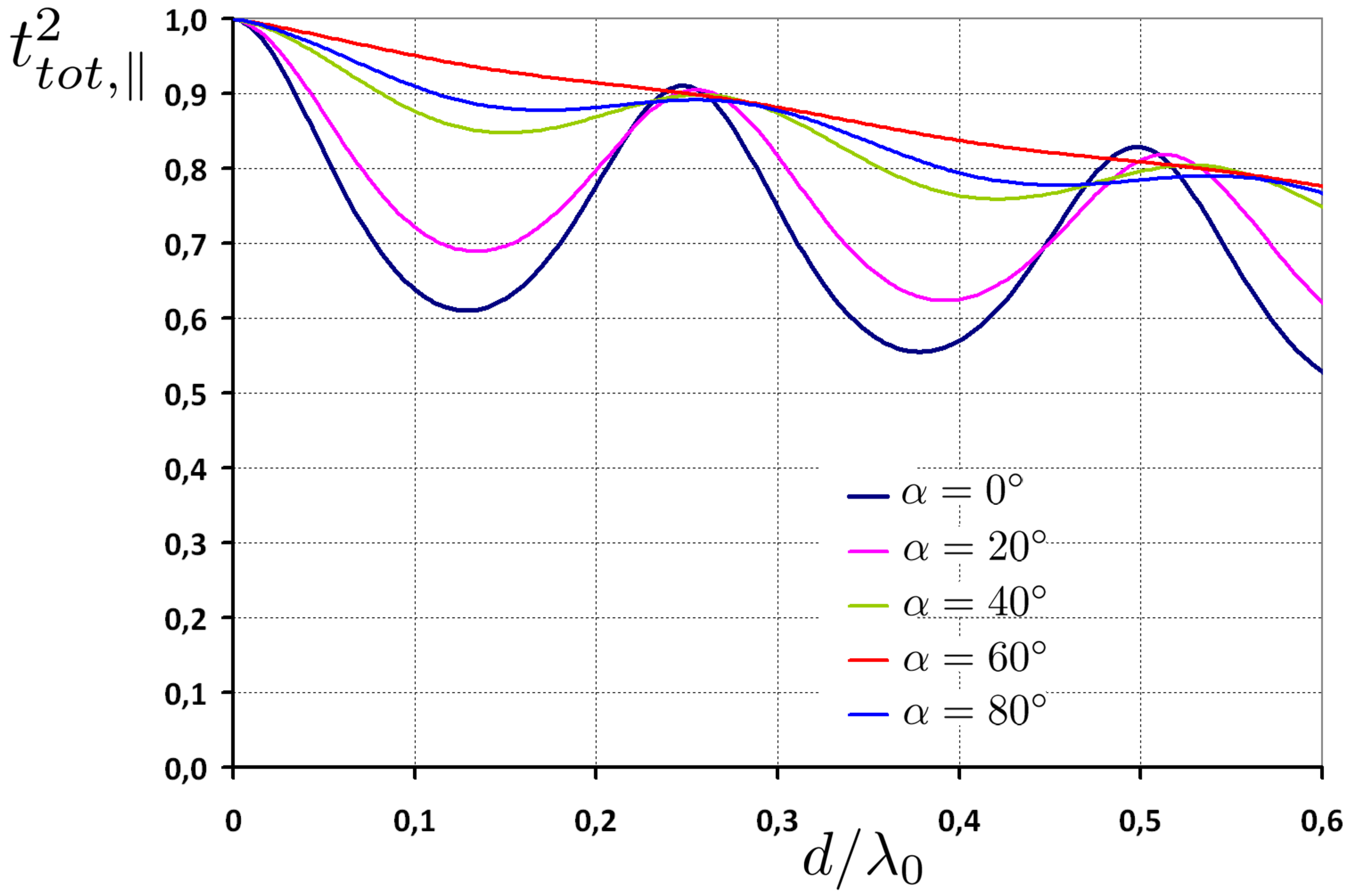
Dielectric Slab with $\varepsilon_r = 3.0, \tan\delta = 0.004$



Dielectric Slab with $\varepsilon_r = 4.0, \tan\delta = 0.03$

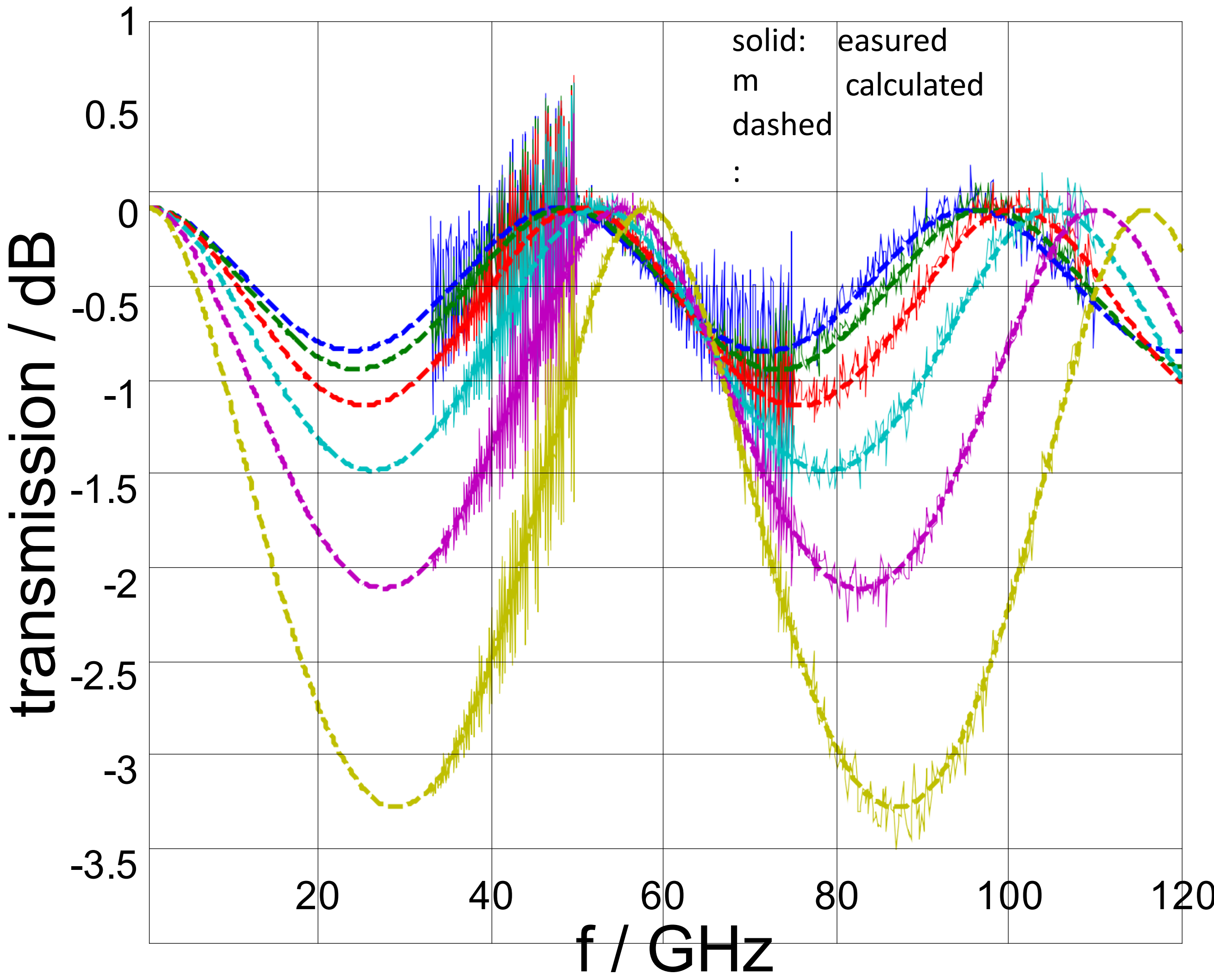


Dielectric Slab with $\varepsilon_r = 4.0, \tan\delta = 0.03$



Dielectric Slab – Permittivity Measurements

Material Characterization (2 mm slab)



curve fitting results → d = 2.08 mm, ε_r = 2.30, tan(δ) = 0.0003

4.6 Problem 6

A dielectric slab has a thickness of 3 mm and a dielectric constant of $\varepsilon_r = 3.2$.

a) Calculate the first two frequencies at which the reflection of a perpendicular polarized incident TEM wave will vanish. The angle of incidence is $0°$.

b) Sketch the path of propagation of the incident, reflected and refracted wave for an angle of incidence of $45°$.

c) At which angle of incidence does the reflection vanish (for any polarization!) at a frequency of 30 GHz and 33 GHz?

Now, assume a certain frequency and a certain angle of incidence.

d) What happens if the dielectric constant increases to $\varepsilon_r = 100$ and finally approaches ∞? What is the angle of refraction and how much energy is transmitted?

$$\lambda_0 = \frac{c}{f}$$

$$\boxed{d = m \cdot \frac{\lambda_0}{2} \cdot \frac{1}{\sqrt{\varepsilon_r - \sin^2\alpha}}, \quad m = 1, 2, \ldots}$$

$d = 3\,mm$; $\varepsilon_r = 3.2$

a) Formula for optimum thickness: $d = m\dfrac{\lambda_0}{2} \cdot \dfrac{1}{\sqrt{\varepsilon_r}}$ (independent of polarization)

$$\lambda_0 = \frac{c_0}{f}$$

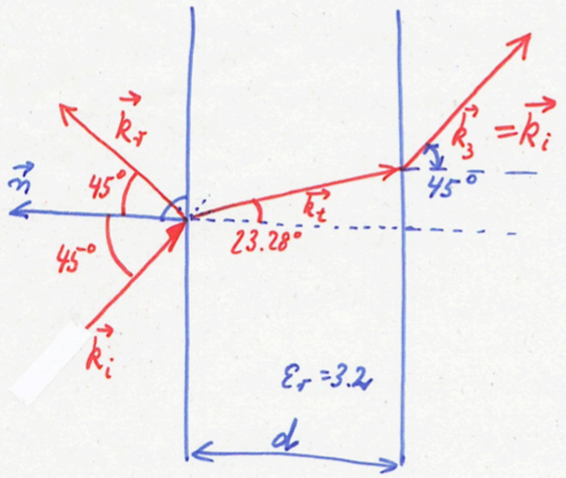
$$\Rightarrow d = m \cdot \frac{c_0}{2f} \cdot \frac{1}{\sqrt{\varepsilon_r}}$$

$$f = m\frac{c_0}{2d} \cdot \frac{1}{\sqrt{\varepsilon_r}} \checkmark$$

$$m = 1: \; f = \frac{3 \cdot 10^8\,m/s}{2 \cdot 3 \cdot 10^{-3}\,m} \cdot \frac{1}{\sqrt{3.2}} = 27.95\,GHz \checkmark$$

$$m = 2: \; f = 2 \cdot 27.95\,GHz = 55.90\,GHz \checkmark$$

b)



Snell's Law:

$$\frac{\sin\alpha_2}{\sin\alpha_1} = \frac{1}{\sqrt{3.2}} = \sqrt{\frac{\varepsilon_1}{\varepsilon_2}}$$

$$\sin\alpha_2 = \frac{1}{\sqrt{3.2}} \underbrace{\sin 45°}_{\frac{1}{\sqrt{2}}} = \frac{1}{\sqrt{6.4}}$$

$$\Rightarrow \alpha_2 = 23.28°$$

sin

4.6 Problem 6

A dielectric slab has a thickness of $3\,\mathrm{mm}$ and a dielectric constant of $\varepsilon_r = 3.2$.

a) Calculate the first two frequencies at which the reflection of a perpendicular polarized incident TEM wave will vanish. The angle of incidence is $0°$.

b) Sketch the path of propagation of the incident, reflected and refracted wave for an angle of incidence of $45°$.

c) At which angle of incidence does the reflection vanish (for any polarization!) at a frequency of $30\,\mathrm{GHz}$ and $33\,\mathrm{GHz}$?

Now, assume a certain frequency and a certain angle of incidence.

d) What happens if the dielectric constant increases to $\varepsilon_r = 100$ and finally approaches ∞? What is the angle of refraction and how much energy is transmitted?

$r_{\parallel} = 0$, $\alpha_i =$ brews---

($r_{\perp} = 0$, Slab eq^n)

c)

$$d = m\frac{\lambda_0}{2}\cdot\frac{1}{\sqrt{\varepsilon_r - \sin^2\alpha_i}} \Rightarrow \sqrt{\varepsilon_r - \sin^2\alpha_i} = \frac{m\lambda_0}{2d}$$

$$\Rightarrow \varepsilon_r - \sin^2\alpha_i = \left(\frac{m\lambda_0}{2d}\right)^2$$

$$\sin^2\alpha_i = \varepsilon_r - \left(\frac{m\lambda_0}{2d}\right)^2$$

$$\sin\alpha_i = \sqrt{\varepsilon_r - \left(\frac{m\lambda_0}{2d}\right)^2} \Rightarrow \alpha_i = \arcsin\sqrt{\varepsilon_r - \left(\frac{m c_0}{2fd}\right)^2}\ ; \text{here } m=1$$

$c_1)$ $f = 30\,\mathrm{GHz} \Rightarrow \alpha_i = 40.53°$

$c_2)$ $f = 33\,\mathrm{GHz} \Rightarrow \alpha_i = 71.98°$

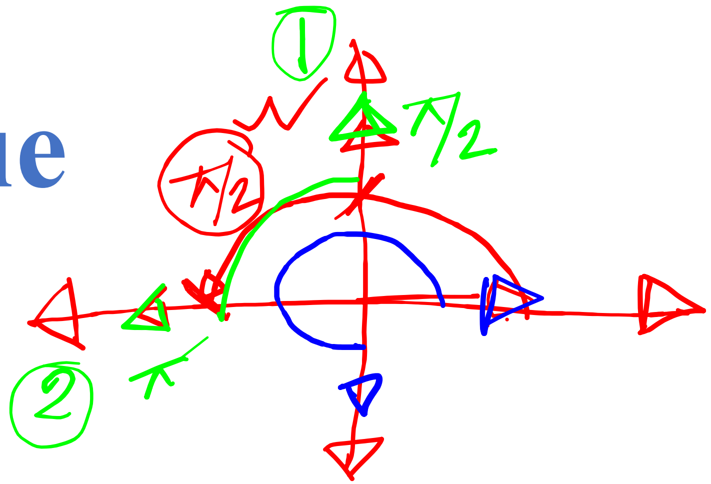
In addition, the reflection does also vanish for parallel polarization (only) if the wave is incident under Brewster's angle (for any frequency). (and for any slab thickness!)

$$\tan\alpha_{Br.} = \sqrt{\varepsilon_r} \Rightarrow \alpha_{Br} = \arctan\sqrt{3.2}$$

$$\approx 60.79°$$

$E_1 = +\pi$ $E_2 = \pi/2$

Polarization Ninja Technique



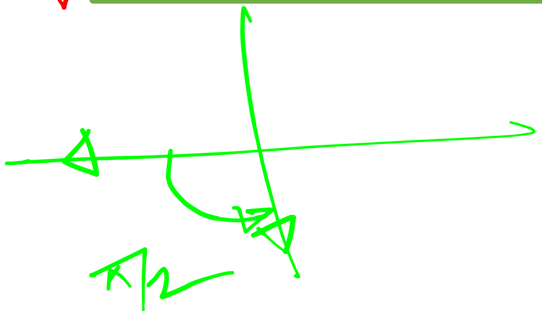
- Make two cosines
- Draw the corresponding vectors
- Start from zero. If it is in between 0 to pi, left handed. If pi to 2pi, right handed.

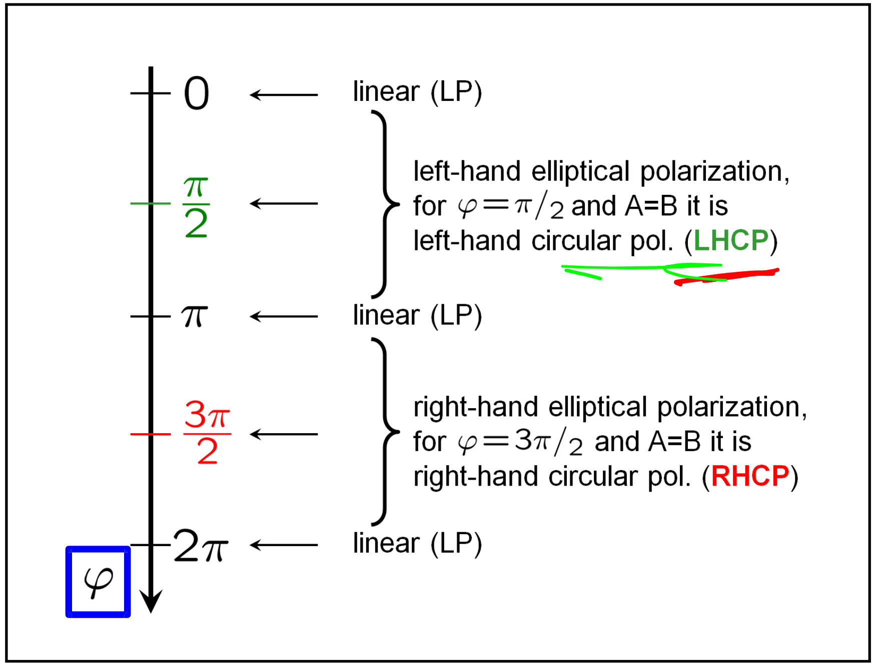
$-\pi/2$

$$\begin{aligned} E_x &= A\cos(\omega t - k_z z) + \pi \\ E_y &= B\cos(\omega t - k_z z + \varphi) \end{aligned}$$

$\pi/2$

$+3\pi/2$





Dielectric Slab

Total reflection of a dielectric slab (e.g., a radome) with thickness d can be avoided by:

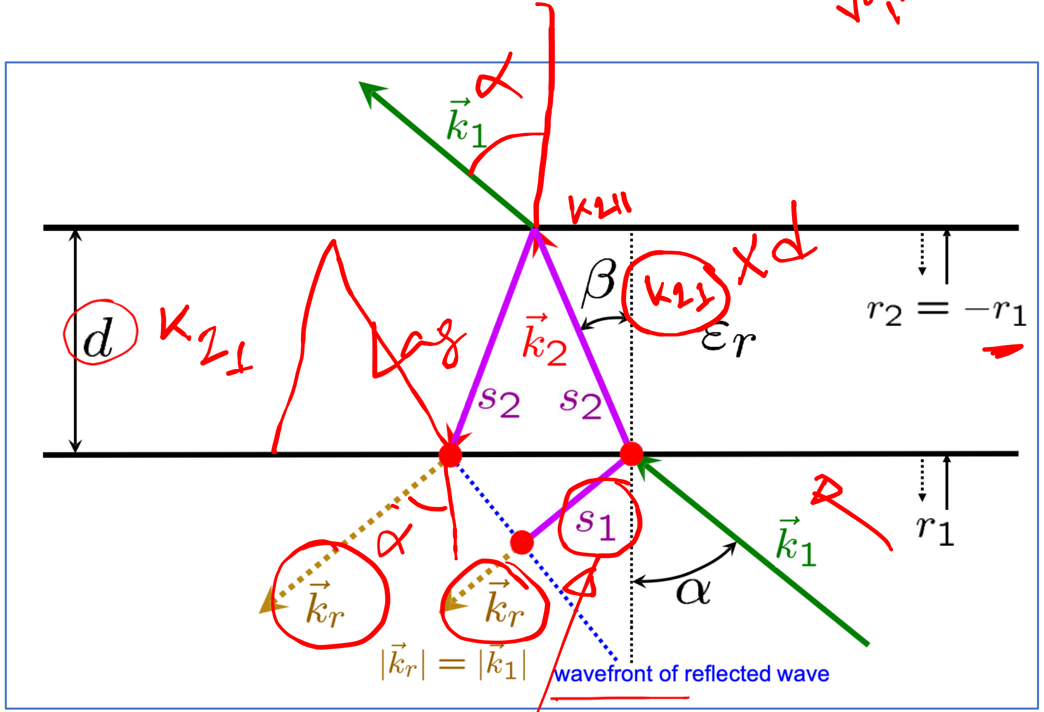
$$\boxed{d = m \cdot \frac{\lambda_0}{2} \cdot \frac{1}{\sqrt{\varepsilon_r - \sin^2\alpha}}, \quad m = 1, 2, \dots}$$

If this condition is fulfilled the power is totally transmitted through the (lossless) dielectric slab.

*** We have seen that at the case of Brewer's Angle!

Dielectric Slab

$\Phi = \_\_\_\_ \ \_\_\_\_$



$$\frac{\sin\alpha_t}{\sin\alpha_i} = \sqrt{1/\varepsilon_r}$$ Lead

The total reflection coefficient of the slab is given by

$r^2 + t^2 = 1$

$$\underline{r}_{tot} = \frac{r_1\left(1 - \mathrm{e}^{-j2\Phi}\right)}{1 - r_1^2 \cdot \mathrm{e}^{-j2\Phi}} \tag{4.129}$$

from which the total transmission coefficient is again calculated with the help of the power balance assuming a lossless slab,

$$\underline{t}_{tot} = \sqrt{1 - \underline{r}_{tot}^2} \tag{4.130}$$

Both total reflection coefficient and total transmission coefficient can be found by analyzing the signal flow through the slab resulting from the multiple reflections and multiple refractions at both interfaces of the slab. The so-called signal flow graph theory based on scattering parameters is applied for this.

Phase Φ equals the phase delay resulting from traveling through the dielectric slab itself,

$$\boxed{\Phi = k_{2\perp} \cdot d} \tag{4.131}$$

The wave number $k_{2\perp}$ takes only into account the perpendicular portion of the refracted wave vector because only the perpendicular and not the parallel portion is reflected,

$$k_{2\perp} = |\vec{k}_{2\perp}| \tag{4.132}$$

From eq. (4.129) we see that the total reflection coefficient $\underline{r}_{tot}$ becomes zero if

$$\mathrm{e}^{-j2\Phi} = 1 \tag{4.133}$$

$\cos 2\phi = 1$

that corresponds to the condition

$$\boxed{\Phi = m \cdot \pi} \quad \text{with} \quad m = 1, 2, 3, \ldots \tag{4.134}$$

$2\phi = 2m\pi$

With eq. (4.131) and condition (4.134) we find the optimum thickness d to avoid reflections and to get total transmission through the slab:

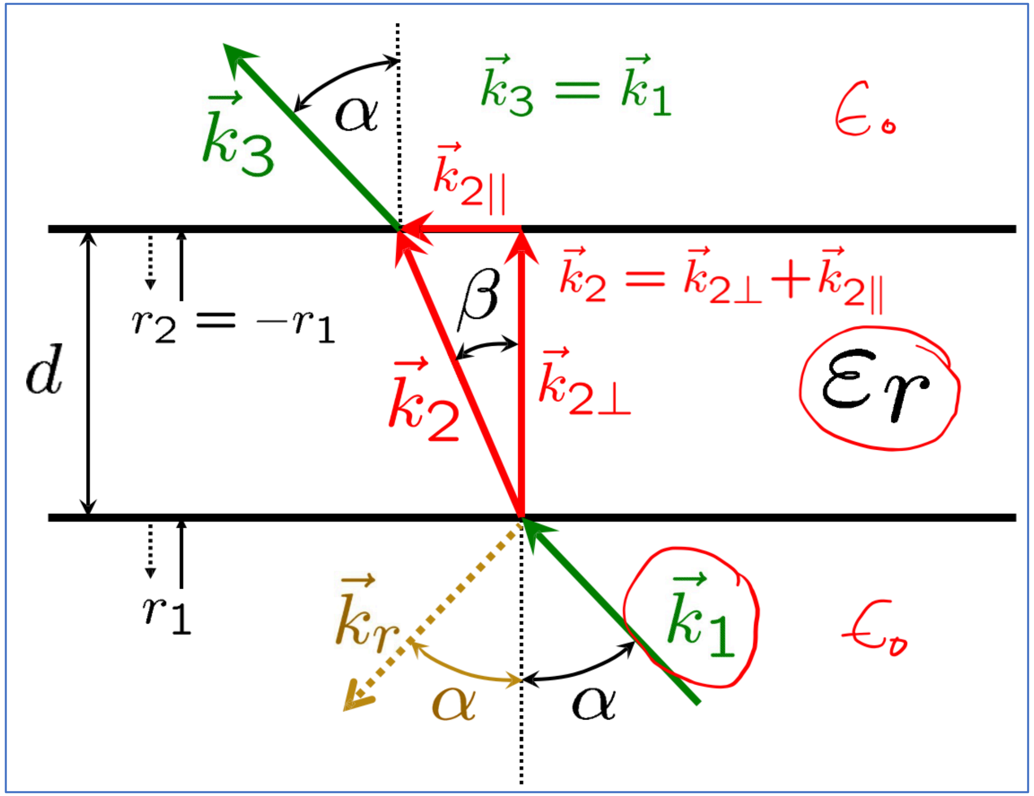
$$\boxed{d = m \cdot \frac{\pi}{k_{2\perp}}} \tag{4.135}$$

$r_1 = r_\perp$

$r_1 = r_\parallel$

Dielectric Slab

$$\boxed{d = m \cdot \frac{\pi}{k_{2\perp}}}$$



With the angle of refraction β we get

$$|\vec{k}_{2\perp}| = |\vec{k}_2| \cos\beta \tag{4.136}$$

The magnitude of wave vector $\vec{k}_2$ is given by the wavelength of the refracted wave as:

$$\begin{aligned} |\vec{k}_2| &= \frac{2\pi}{\lambda_2} \\ &= \frac{2\pi}{\lambda_0}\sqrt{\varepsilon_r} \end{aligned} \tag{4.137}$$

where λ_0 is the wavelength of the incident wave in free space (or in air).
For $|\vec{k}_{2\perp}|$ we now obtain

$$\begin{aligned} |\vec{k}_{2\perp}| &= |\vec{k}_2| \cos\beta \\ &= \frac{2\pi}{\lambda_0}\sqrt{\varepsilon_r}\cos\beta \end{aligned} \tag{4.138}$$

$\sin^2\alpha = \epsilon_r \sin^2\beta$
$= \epsilon_r -$

and the condition of eq. (4.135) reads

$$d = m \cdot \frac{\lambda_0}{2\sqrt{\varepsilon_r}\cos\beta} \tag{4.139}$$

With Snell's law

$$\frac{\sin\alpha}{\sin\beta} = \sqrt{\varepsilon_r} \tag{4.140}$$

we finally get the thickness d of the slab for total transmission (i.e., no reflection) with respect to the angle of incidence α

$$\boxed{d = m \cdot \frac{\lambda_0}{2} \cdot \frac{1}{\sqrt{\varepsilon_r - \sin^2\alpha}}, \qquad m = 1, 2, \ldots} \tag{4.141}$$

4.6 Problem 6

A dielectric slab has a thickness of $3\,\mathrm{mm}$ and a dielectric constant of $\varepsilon_r = 3.2$.

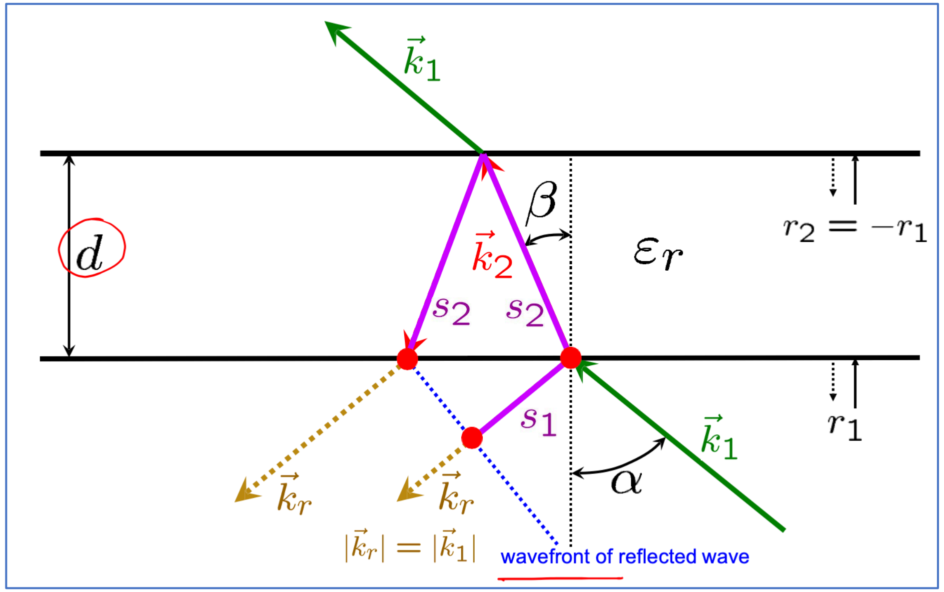
a) Calculate the first two frequencies at which the reflection of a perpendicular polarized incident TEM wave will vanish. The angle of incidence is $0°$.

b) Sketch the path of propagation of the incident, reflected and refracted wave for an angle of incidence of $45°$.

c) At which angle of incidence does the reflection vanish (for any polarization!) at a frequency of $30\,\mathrm{GHz}$ and $33\,\mathrm{GHz}$?

Now, assume a certain frequency and a certain angle of incidence.

d) What happens if the dielectric constant increases to $\varepsilon_r = 100$ and finally approaches ∞? What is the angle of refraction and how much energy is transmitted?



d) For greater and greater ε_r the angle of refraction tends to $0°$ and the transmitted energy will finally approach zero.
The amount of transmitted power can be calculated with the help of the total reflection and transmission coefficients.

$$r_{tot} = \frac{r(1-e^{-j2\Phi})}{1-r^2 e^{-j2\Phi}} \Rightarrow |t_{tot}| = \sqrt{1-|r_{tot}|^2}$$

transmitted power: $|t_{tot}|^2 = 1 - |r_{tot}|^2$

with $r = r_{\parallel}$ or $r = r_{\perp}$ of the single air-to-dielectric interface.

$$\boxed{r_{tot} = \frac{r_1\left(1 - e^{-j2\Phi}\right)}{1 - r_1^2 \cdot e^{-j2\Phi}}}$$

$$\boxed{t_{tot} = \sqrt{1 - r_{tot}^2}}$$

Reflection and Refraction

cos(A-B) =

4.6.3 Metallic, non-magnetic medium (medium no. 2):

$$r_{\parallel} = \frac{E_{r0\parallel}}{E_{i0\parallel}} = \frac{Z_2\cos\alpha_2 - Z_1\cos\alpha_1}{Z_2\cos\alpha_2 + Z_1\cos\alpha_1}$$ **parallel polarization**

cos

$$t_{\parallel} = \frac{2\sqrt{Z_1 Z_2}\cos\alpha_1\cos\alpha_2}{Z_1\cos\alpha_1 + Z_2\cos\alpha_2}$$ **parallel polarization**

sin

$$r_{\perp} = \frac{E_{r0\perp}}{E_{i0\perp}} = \frac{Z_2\cos\alpha_1 - Z_1\cos\alpha_2}{Z_2\cos\alpha_1 + Z_1\cos\alpha_2}$$ **perpendicular polarization**

$$t_{\perp} = \frac{2\sqrt{Z_1 Z_2}\cos\alpha_1\cos\alpha_2}{Z_2\cos\alpha_1 + Z_1\cos\alpha_2}$$ **perpendicular polarization**

$$Z_2 = \sqrt{\frac{\mu_0}{\underline{\varepsilon}}} \to 0$$

100

$$\underline{\varepsilon} = \varepsilon - j\frac{\sigma}{\omega} \approx -j\frac{\sigma}{\omega}$$

$$r_{\perp} = r_{\parallel} = -1$$

$$t_{\perp} = t_{\parallel} = 0$$

The incident wave is totally reflected.

4.6 Problem 6

A dielectric slab has a thickness of $3\,\mathrm{mm}$ and a dielectric constant of $\varepsilon_r = 3.2$.

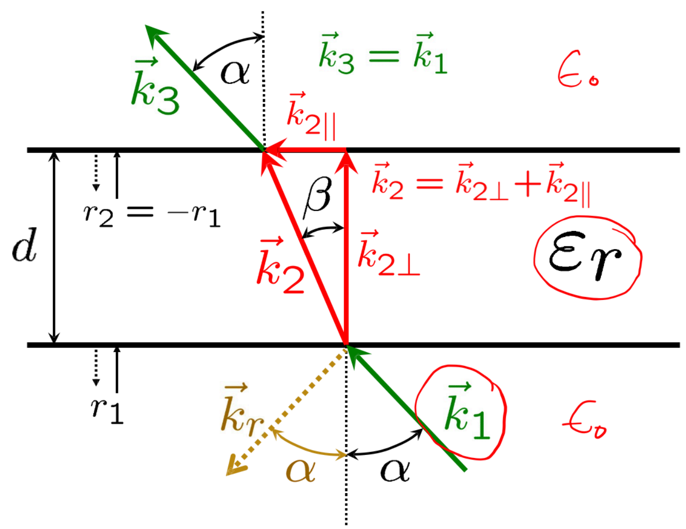
a) Calculate the first two frequencies at which the reflection of a perpendicular polarized incident TEM wave will vanish. The angle of incidence is $0°$.

b) Sketch the path of propagation of the incident, reflected and refracted wave for an angle of incidence of $45°$.

c) At which angle of incidence does the reflection vanish (for any polarization!) at a frequency of $30\,\mathrm{GHz}$ and $33\,\mathrm{GHz}$?

Now, assume a certain frequency and a certain angle of incidence.

d) What happens if the dielectric constant increases to $\varepsilon_r = 100$ and finally approaches ∞? What is the angle of refraction and how much energy is transmitted?



$$\begin{aligned} |\vec{k}_{2\perp}| &= |\vec{k}_2| \cos\beta \\ &= \frac{2\pi}{\lambda_0}\sqrt{\varepsilon_r}\cos\beta \end{aligned}$$

$$\Phi = k_{2\perp} \cdot d$$

$\phi = k_{2\perp} \cdot d$

$k_{2\perp} = \frac{2\pi}{\lambda_0}\sqrt{\varepsilon_r}\cos\beta$ with $\cos\beta = \sqrt{1-\sin^2\beta}$

and $\frac{\sin\beta}{\sin\alpha} = \frac{1}{\sqrt{\varepsilon_r}}$ (Snell's Law)

(β is the angle of refraction)

for example: $\alpha = 45°$; $\varepsilon_r = 100 \Rightarrow \beta = 4.05°$

" ; $\varepsilon_r = 1000 \Rightarrow \beta = 1.28°$