Lecture 10

Propagation, Dispersion and Homogeneous Waves

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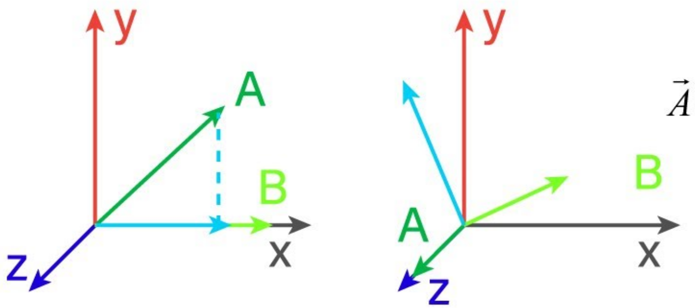
Use of Calculator

VECTOR REVIEW:

DOT PRODUCT & CROSS PRODUCT



$$\vec{A}\times\vec{B} = \left|\begin{pmatrix} \hat{i} & \hat{j} & \hat{k} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{pmatrix}\right|$$

$$= \left|\vec{A}\right|\left|\vec{B}\right| \sin\theta \hat{e}$$

$$\vec{A}\cdot\vec{B} = A_x B_x + A_y B_y + A_z B_z$$

$$= \left|\vec{A}\right|\left|\vec{B}\right| \cos\theta$$

a. $\langle 4, 5\rangle \cdot \langle 2, 3\rangle = 4(2) + 5(3)$

$= 8 + 15$

$= 23$

$$x_1 \times x_2 = \begin{vmatrix} i & j & k \\ 2 & -3 & 1 \\ -2 & 1 & 1 \end{vmatrix}$$

$$= \begin{vmatrix} -3 & 1 \\ 1 & 1 \end{vmatrix} i - \begin{vmatrix} 2 & 1 \\ -2 & 1 \end{vmatrix} j + \begin{vmatrix} 2 & -3 \\ -2 & 1 \end{vmatrix} k$$

$$= [(-3)(1)-(1)(1)]i - [(2)(1)-(-2)(1)]j + [(2)(1)-(-2)(-3)]k$$

$$= -4i - 4j + 8k$$

4.3 Problem 3

A $5\,\text{GHz}$ plane wave propagates in a dielectric material characterized by $\varepsilon_r = 2.53$, $\mu_r = 1$, and $\sigma = 0\,\frac{\text{S}}{\text{m}}$. The electric vector field of this wave is $\vec{E} = 10\,\frac{\text{V}}{\text{m}}\cos(\omega t - kz)\,\vec{e}_x$.

a) Determine the phase velocity v_{ph}, the wavelength λ , and the wave number k.

b) Write down the time-domain expression of the magnetic field strength $\vec{H}$.

Now, the propagating wave impinges perpendicularly on a large sheet of gold ($\sigma = 4.1 \times 10^7\,\text{S/m}$).

c) What is the depth at which the wave's amplitude is reduced to $2\,\%$ of its initial value on the surface?

d) Determine the surface current vector field $\vec{J}_s$.

$$v_{ph} = \frac{1}{\sqrt{\mu\epsilon}} \quad , \quad \lambda = \frac{v_{ph}}{f} \quad , \quad k = \frac{2\pi}{\lambda}$$

$f = 5\,GHz$, $\varepsilon_r = 2.53$, $\mu_r = 1$, $\sigma = 0$

$\vec{E} = 10\frac{V}{m}\cos(2\pi f t - kz)\,\vec{e}_x$

a) $$v_{ph} = \frac{1}{\sqrt{\varepsilon\mu}} = \frac{1}{\sqrt{\varepsilon_0\varepsilon_r \cdot \mu_0\mu_r}} = \frac{c_0}{\sqrt{\varepsilon_r\mu_r}} \approx 1.88476 \times 10^8\,\frac{m}{s}$$

$$\approx 188477\,\frac{km}{s}$$

$$\lambda = \frac{v_{ph}}{f} = 37.69\,mm$$

$$k = \frac{2\pi}{\lambda} = 166.68\,\frac{1}{m}$$

$$k = \frac{2\pi}{\lambda} =$$

$$v_{ph} = \frac{1}{\sqrt{\mu\epsilon}}$$

4.3 Problem 3

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$\vec{H} = H_0 \cos(\omega t - kz)\,\hat{e}_y$

$\vec{H}^D(t)$ = time domain

$\vec{H}^D(\omega)$ = frequency

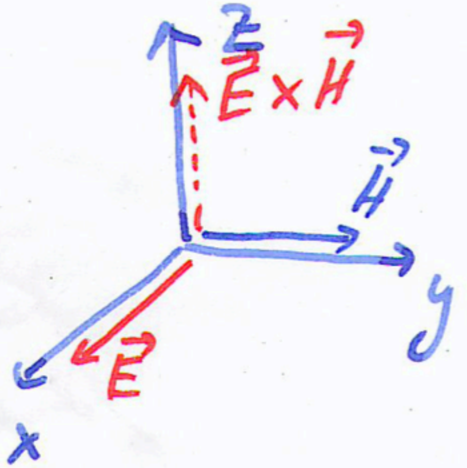
$e^{j\omega}$

$f = 5\,\text{GHz}$, $\varepsilon_r = 2.53$, $\mu_r = 1$, $\sigma = 0$

$\vec{E} = 10\,\frac{\text{V}}{\text{m}}\cos(2\pi f t - kz)\,\vec{e}_x$

$Z_F = \sqrt{\frac{\mu}{\varepsilon}} = \sqrt{\frac{\mu_0}{\varepsilon_r \varepsilon_0}}$

b)



$$\vec{H} = H_y\,\vec{e}_y = \frac{E_0}{Z}\cos(2\pi f t - kz)\,\vec{e}_y$$

$$\text{with } \boxed{Z = \sqrt{\frac{\mu}{\varepsilon}} = \sqrt{\frac{\mu_0}{\varepsilon_0}}\cdot\frac{1}{\sqrt{\varepsilon_r}} = \frac{Z_0}{\sqrt{\varepsilon_r}}}$$

$$\approx \frac{120\pi\,\Omega}{\sqrt{2.53}} \approx 237\,\Omega$$

$$\Rightarrow \vec{H} = 0.042\,\frac{\text{A}}{\text{m}}\cos(2\pi f t - kz)\,\vec{e}_y$$

4.3 Problem 3

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c) What is the depth at which the wave's amplitude is reduced to 2 % of its initial value on the surface?

20dB

10dB

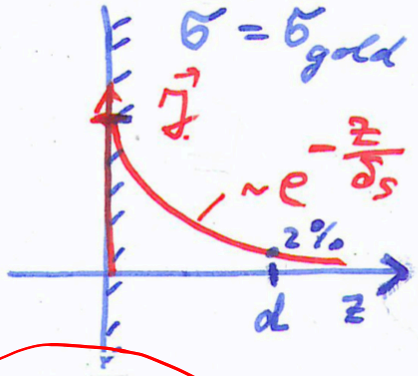
d) Determine the surface current vector field $\vec{J}_s$.

$\delta_s = \sqrt{\frac{2}{\omega\mu\sigma}}$, e^{-x/δ_s} change at amplitude

$f = 5\,\mathrm{GHz}$, $\varepsilon_r = 2.53$, $\mu_r = 1$, $\sigma = 0$

$\vec{E} = 10\frac{\mathrm{V}}{\mathrm{m}}\cos(2\pi f t - kz)\,\vec{e}_x$

c)



Skin depth

f	δ_s
50 Hz	9.4 mm
1 kHz	2.1 mm
1 MHz	66 μm
1 GHz	2.1 μm
100 GHz	0.21 μm

$|\vec{J}| \sim e^{-\frac{z}{\delta_s}}$ with skin depth δ_s

$$\delta_s = \sqrt{\frac{2}{\omega\mu\sigma}} = 1.11\,\mu\mathrm{m}$$

$$\left[\, e^{-\frac{d}{\delta_s}} = \frac{2}{100} \,\right]$$

$$-\frac{d}{\delta_s} = \ln\frac{2}{100}$$

$$d = -\delta_s \ln\frac{2}{100} = \delta_s \cdot \ln 50$$

$$\approx 4.34\,\mu\mathrm{m}$$

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$$\vec{J}_s = \vec{n}_{12} \times (\vec{H}_2 - \vec{H}_1)$$

$f = 5\,GHz$, $\varepsilon_r = 2.53$, $\mu_r = 1$, $\sigma = 0$

$\vec{E} = 10\frac{V}{m}\cos(2\pi f t - kz)\,\vec{e}_x$

4.3d) (19)

$|\vec{J}_s| = |\vec{J}_{max}| \cdot \delta_s$

$\vec{J}_s = \vec{n}_{12} \times (\vec{H}_2 - \vec{H}_1)$

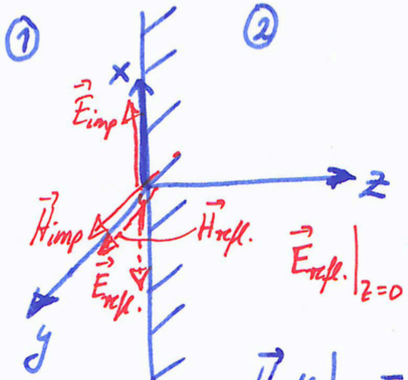
Approximation in case of very good conductor: $\vec{H}_2 \approx 0$

$\vec{J}_s = -\vec{n}_{12} \times \vec{H}_1$

$\vec{H}_1$ consists of impinging wave $\vec{H}_{imp}$ plus reflected wave $\vec{H}_{refl.}$:

$\vec{H}_1 = \vec{H}_{imp} + \vec{H}_{refl.}$

with $\vec{H}_{imp} = H_0 \cos(\omega t - kz)\,\vec{e}_y$ and $H_0 = 0.042\frac{A}{m}$



$\vec{E}_{refl.}|_{z=0} = -\vec{E}_{imp}|_{z=0}$ because $\vec{E}_{tan} = 0$ ($\vec{E}_{tan} = \vec{E}_{imp} + \vec{E}_{refl.}$)

$\vec{H}_{refl.}|_{z=0} = \vec{H}_{imp}|_{z=0}$

$\vec{H}_{refl.} = H_0 \cos(\omega t + kz)\,\vec{e}_y$

$\Rightarrow \vec{H}_1|_{z=0} = 2H_0 \cos(\omega t)\,\vec{e}_y$; $\vec{H}_1 = H_0\{\cos(\omega t - kz) + \cos(\omega t + kz)\}\,\vec{e}_y$

$$\vec{J}_s = -\begin{pmatrix}0\\0\\1\end{pmatrix} \times \begin{pmatrix}0\\2H_0\cos(\omega t)\\0\end{pmatrix} = 2H_0\cos(\omega t)\,\vec{e}_x$$

Dispersion due to Losses (Complex Wave Vector)

$$\underline{\varepsilon} = \varepsilon\left(1 - j\frac{\sigma}{\varepsilon\,\omega}\right) = \varepsilon\left(1 - j\frac{1}{\tau_{\text{relax}}\,\omega}\right)$$

Complex permit--

$k = \omega\sqrt{\mu\varepsilon}$

$$\underline{k} = \omega\sqrt{\mu\varepsilon - j\frac{\mu\sigma}{\omega}}$$

or its square

$$\underline{k}^2 = \omega^2\mu\varepsilon - j\omega\mu\sigma$$

We can separate $\underline{k}$ into its real and imaginary parts:

$$\underline{k} = k' - jk''$$

$$\underline{k}^2 = (k' - jk'')^2 = k'^2 - k''^2 - j2k'k''$$

$k = k' - jk''$

$(\omega t - kx - ky)$

$x_1 x_2 =$ -- $\rightarrow$

$$k'k'' = \frac{\omega\mu\sigma}{2} = \frac{1}{\delta^2}$$

$x_1^2 - x_2^2 =$ -- $\rightarrow$

$$k'^2 - k''^2 = \omega^2\mu\varepsilon$$

The physical solutions of these relations are

$$k' = \omega\sqrt{\frac{\mu\varepsilon}{2}}\left(\sqrt{1+\left(\frac{1}{\omega\,\tau_{\text{relax}}}\right)^2}+1\right)^{1/2}$$

$$k'' = \omega\sqrt{\frac{\mu\varepsilon}{2}}\left(\sqrt{1+\left(\frac{1}{\omega\,\tau_{\text{relax}}}\right)^2}-1\right)^{1/2}$$

$(\tau_{\text{relax}} = \varepsilon/\sigma)$

With the loss tangent

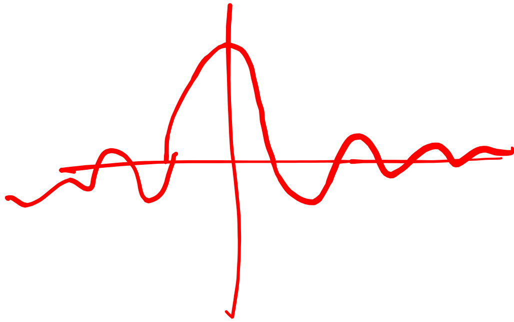
$$\tan\delta = \frac{\sigma}{\omega\varepsilon}$$

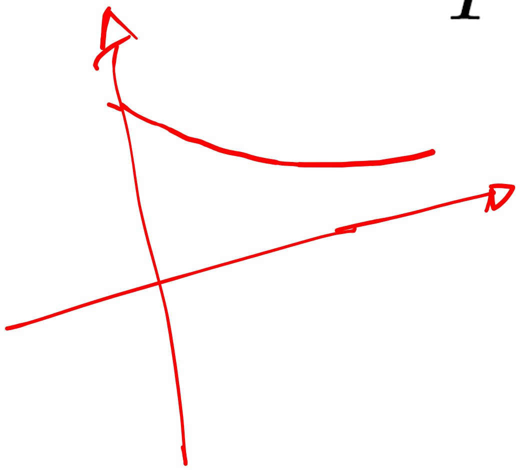
- The factor **k′** is called **phase constant** ; because k′ contributes to the phase of the wave
- The factor **k′′** is called **damping constant** ; because it causes a damping of the wave

Lossy Media

Waves in **lossy** media → complex wave number $\underline{k}$

→ **damping** & **dispersion** !!

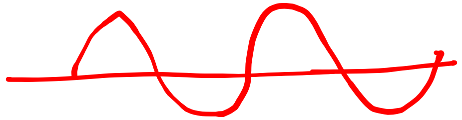




$$
\begin{aligned}
F(z,t) &= A \cdot \Re\{ e^{j(\omega t - \underline{k}\, z)} \} \\
&= A \cdot \Re\{ e^{j(\omega t - (k' - jk'')\, z)} \} \\
&= A \cdot \Re\{ e^{j(\omega t - k'\, z)} e^{-k''\, z} \} \\
&= A\, e^{-k''\, z} \cos(\omega t - k' z)
\end{aligned}
$$

❖ The factor **k″** is called **damping constant** ; because it causes a damping of the wave

Dispersion due to Losses



Phase Velocity: $\approx$ wave velocity

$$v_{ph} = \frac{\omega}{k'}$$

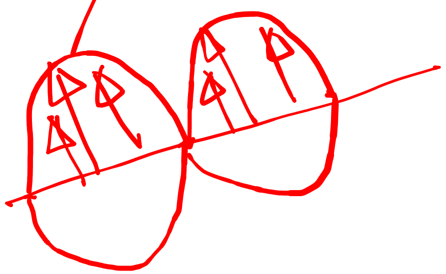
$$= \sqrt{\frac{2}{\mu\varepsilon}}\left(\sqrt{1+\left(\frac{1}{\omega\,\tau_{\text{relax}}}\right)^2}+1\right)^{-1/2}$$

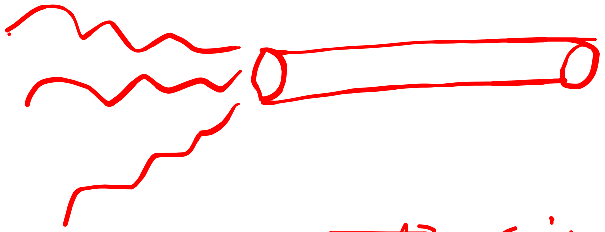
$$\tau_{\text{relax}} = \frac{\varepsilon}{\sigma}$$

❖ The **phase velocity** of a wave is **the rate at which the wave propagates** in some medium. This is the velocity at which the phase of any one frequency component of the wave travels.

Dispersion due to Losses

envelope velocity





$\rightarrow \sin(\omega t - kz)$

$\rightarrow \sin(\omega t - kz + \pi/2)$

Group Velocity:

$$v_g = \frac{d\omega}{dk'}$$

$$= \frac{v_{ph}}{1 - \frac{\omega}{v_{ph}} \frac{d v_{ph}}{d\omega}}$$

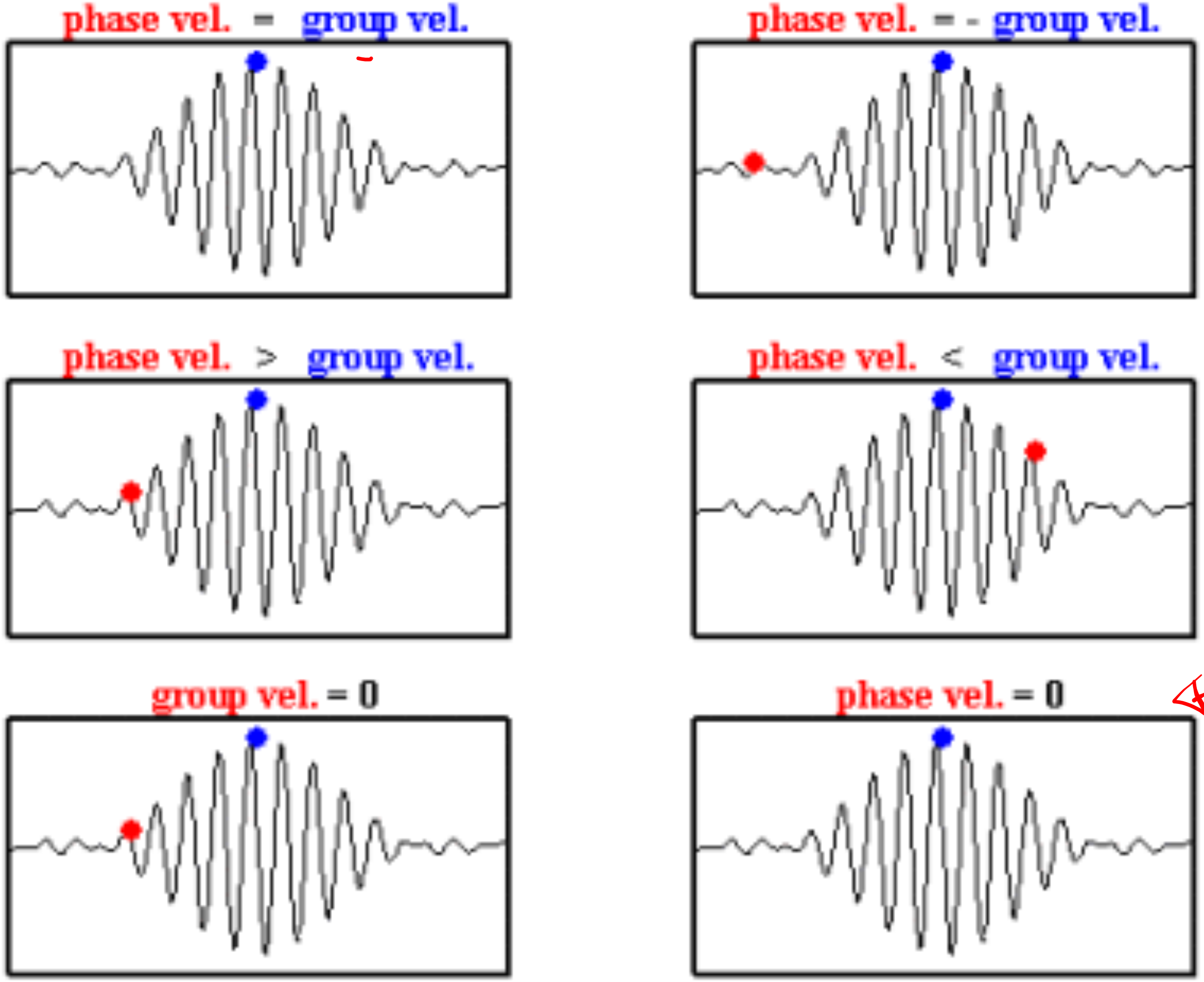
Amplitude Modulation

❖ The **group velocity** of a wave is the velocity with which the **overall envelope** shape of the wave's amplitudes—known as the **modulation or envelope** of the wave—propagates through space.

Phase Velocity Vs Group Velocity

$V_{ph} = V_g$

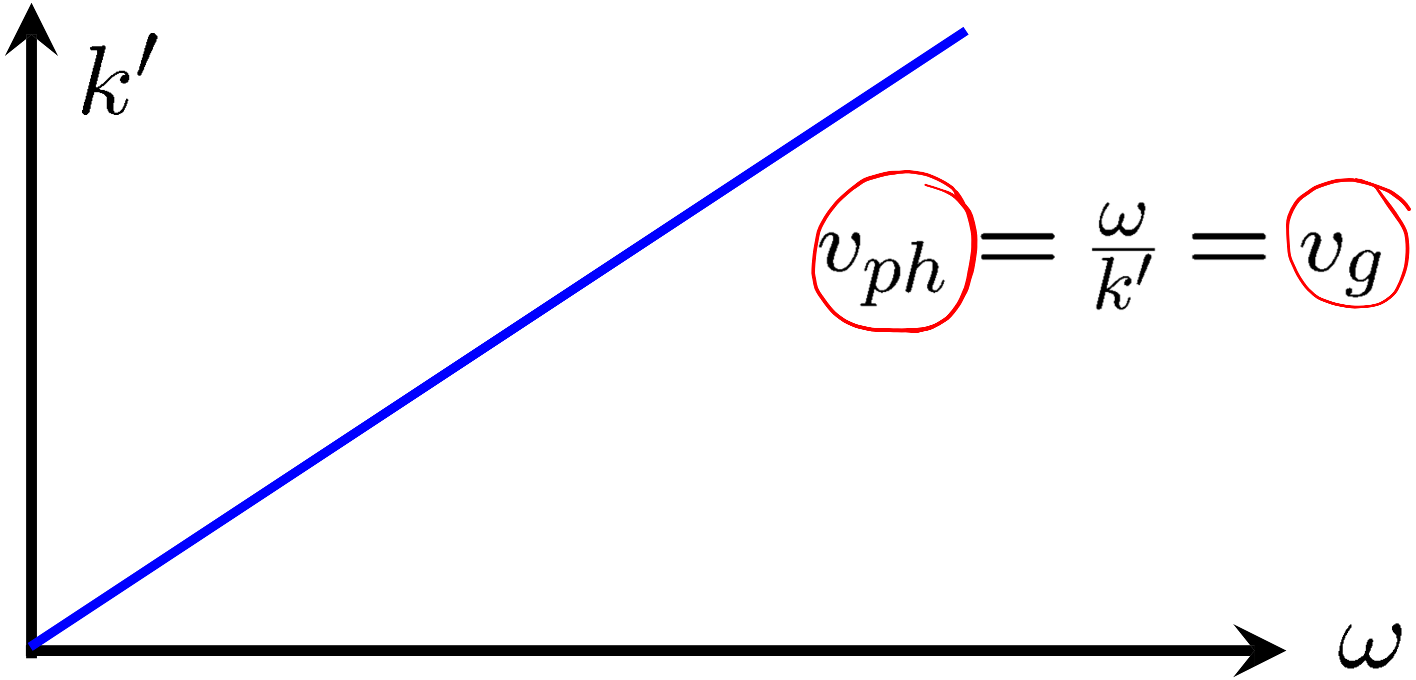
$V_{ph} > V_g$



isvr

Group and Phase Velocity

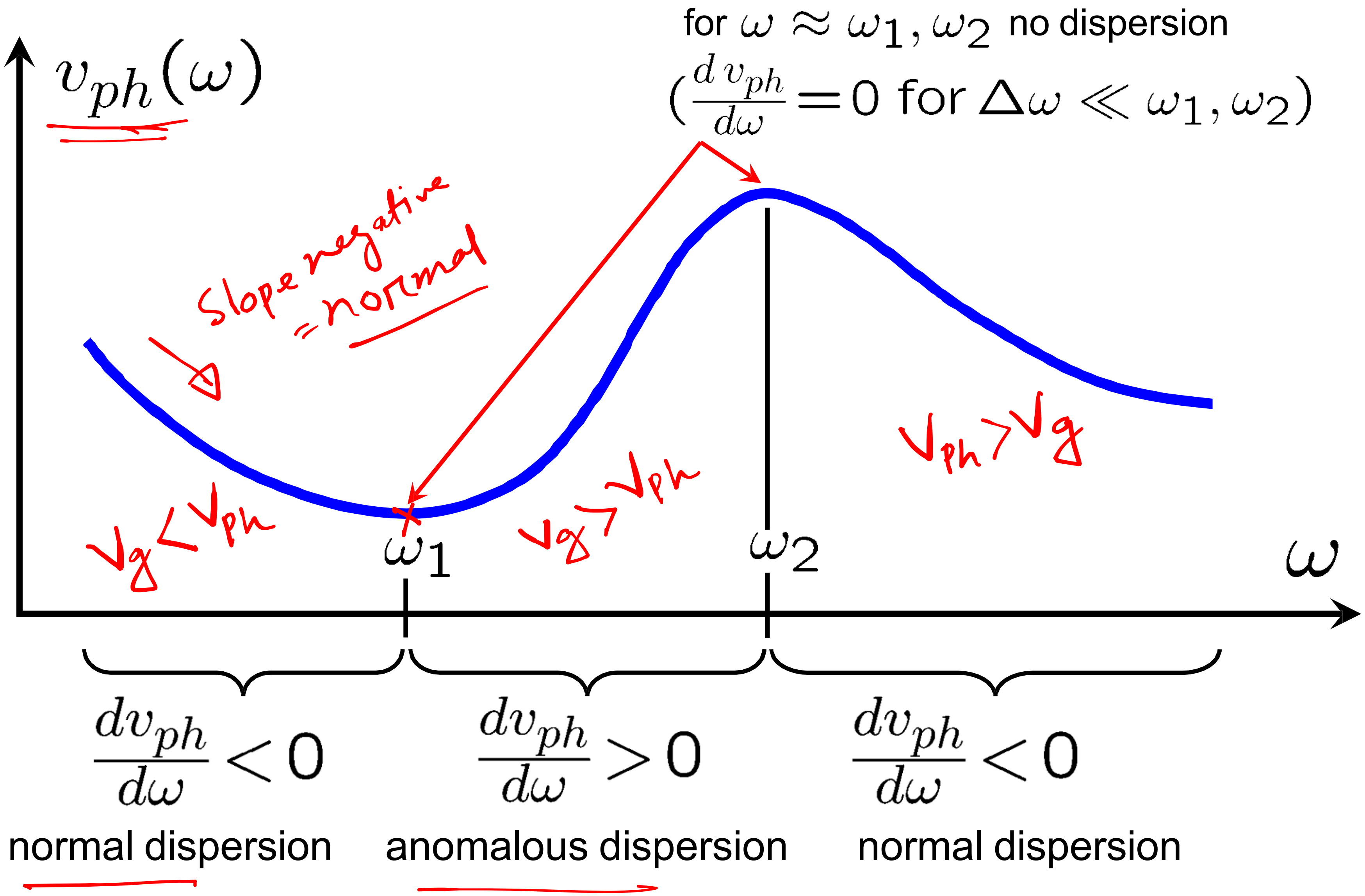
NO Dispersion



$$v_{ph} = \frac{\omega}{k'} = v_g = \frac{1}{\sqrt{\mu\varepsilon}}$$

❖ In dispersion free media the group velocity equals to the phase velocity.

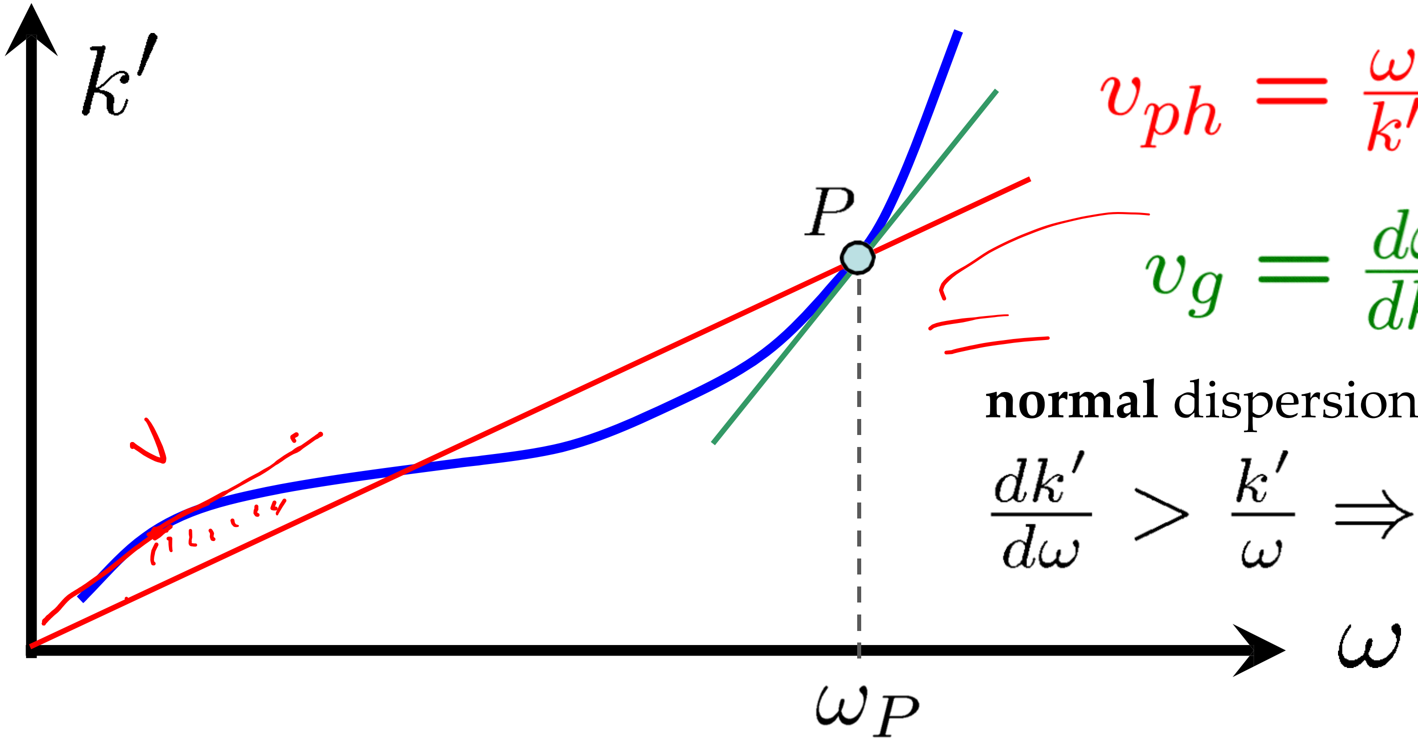
Dispersion



In case of **normal dispersion**, the group velocity is lower than the phase velocity (**Vg < Vph**). In case of **anomalous dispersion**, the group velocity is greater than the phase velocity (**Vg > Vph**).

Group and Phase Velocity

Normal Dispersion at $\omega \approx \omega_P$



$$v_{ph} = \frac{\omega}{k'} = \frac{1}{k'/\omega}$$

$$v_g = \frac{d\omega}{dk'} = \frac{1}{dk'/d\omega}$$

normal dispersion (in point P):

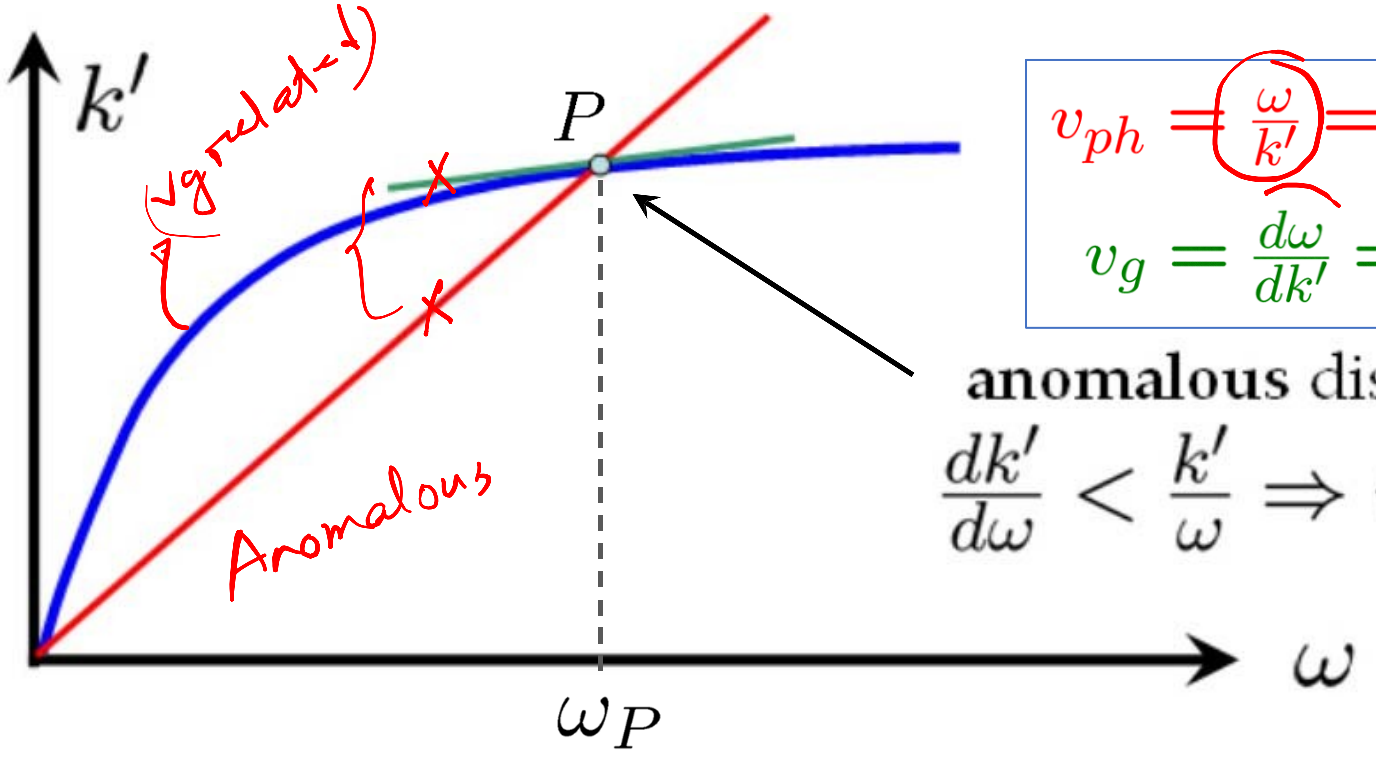
$$\frac{dk'}{d\omega} > \frac{k'}{\omega} \Rightarrow v_g < v_{ph}$$

In case of **normal dispersion**, the group velocity is lower than the phase velocity (**Vg < Vph**).

Group and Phase Velocity

$\boxed{y = mx}$

Anomalous Dispersion at $\omega \approx \omega_P$



$$v_{ph} = \frac{\omega}{k'} = \frac{1}{k'/\omega}$$

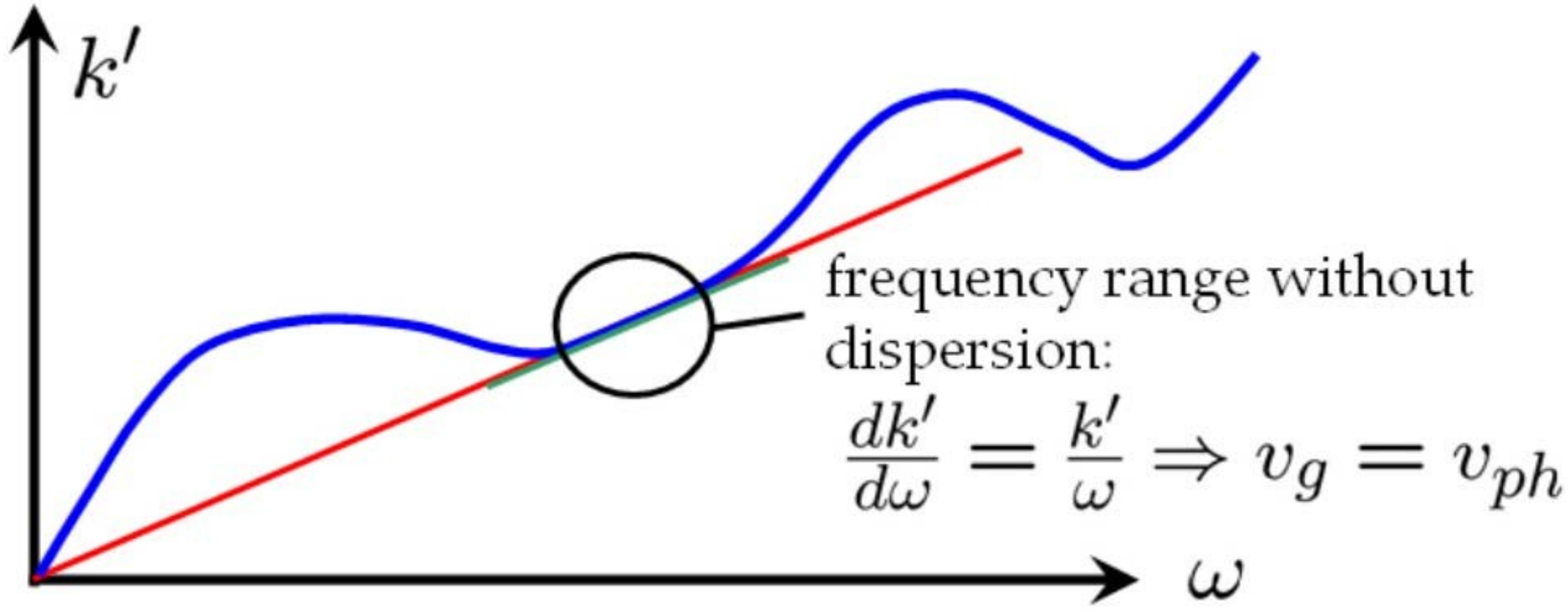
$$v_g = \frac{d\omega}{dk'} = \frac{1}{dk'/d\omega}$$

anomalous dispersion:

$$\frac{dk'}{d\omega} < \frac{k'}{\omega} \Rightarrow v_g > v_{ph}$$

In case of **anomalous dispersion**, the group velocity is greater than the phase velocity (**Vg > Vph**).

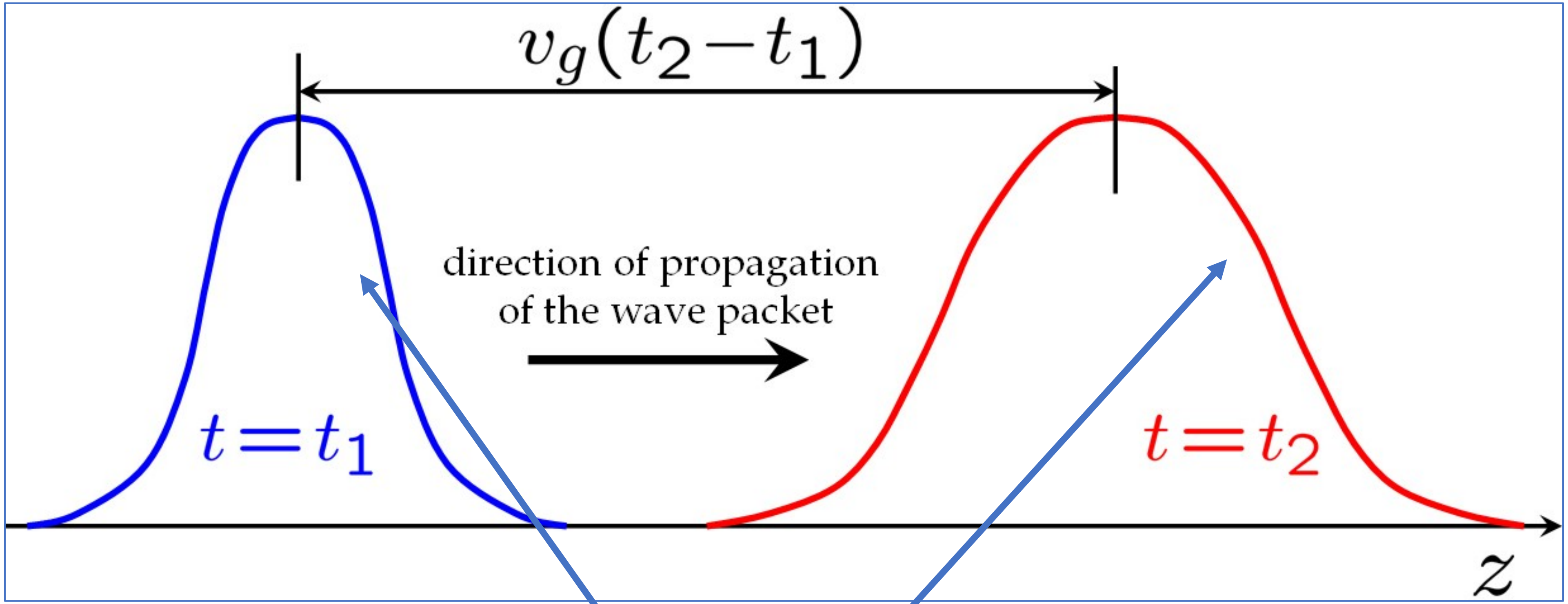
Free of Dispersion



Wave Packet Propagation in Dispersive Media

Broadening of the wave packet due to dispersion.

→ Reduction of Data Rate, higher BER (Bit Error Rate), etc.



The **Blue Curve** is less scattered (dispersed) than the **Red Curve.**

4.1 Problem 1

A harmonic and plane TEM wave with the frequency of 20 GHz and the amplitude of the electric field vector of $1\,\frac{\mathrm{V}}{\mathrm{m}}$ propagates in negative z-direction (Cartesian coordinate system) in a lossless, homogeneous, linear and isotropic medium with a relative dielectric constant of $\varepsilon_r = 2$ and $\mu_r = 1$.

a) Write down the four Maxwell's equations in differential notation

   i) for the general case (non harmonic time dependent fields) and

   ii) in complex notation (harmonic time dependent fields).

b) Give the wavelength λ.

c) Give the wavevector $\vec{k}$.

d) Calculate the wave impedance Z_F.

e) Give the phase velocity v_{ph}.

f) Which field defines the state of polarization? What different kinds of polarization do exist?

g) Formulate the electric and magnetic field components for a linear polarization in x-direction.

h) Compute the power flow through an area of 2.5 m^2 perpendicular to the direction of propagation.

$$\begin{aligned}
\text{curl}\,\vec{H} &= \vec{J} + \frac{\partial \vec{D}}{\partial t} \\
\text{curl}\,\vec{E} &= -\frac{\partial \vec{B}}{\partial t} \\
\text{div}\,\vec{D} &= \varrho \\
\text{div}\,\vec{B} &= 0
\end{aligned}$$

$$\begin{aligned}
\text{curl}\,\underline{\vec{H}} &= \underline{\vec{J}} + j\omega\underline{\vec{D}} \\
\text{curl}\,\underline{\vec{E}} &= -j\omega\underline{\vec{B}} \\
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$$c = \frac{c_0}{\sqrt{\mu_r \epsilon_r}}$$

$$\lambda = \frac{c}{f}$$

(b) $$c = \frac{c_0}{\sqrt{\mu_r \epsilon_r}} = \frac{3\times10^8}{\sqrt{1\times 2}} = 2.12\times10^8\ \mathrm{ms^{-1}}$$

$$\lambda = \frac{c}{f} = \frac{2.12\times10^8}{20\times10^9}\ \mathrm{m} = 0.0106\ \mathrm{m}$$

(c) $$\vec{k} = \frac{2\pi}{\lambda}(-\hat{e}_z) = \frac{2\pi}{0.0106}(-\hat{e}_z) = -592.7533\,\hat{e}_z$$

$$\vec{k} = \begin{pmatrix} 0 \\ 0 \\ -592.7533 \end{pmatrix}$$

(d) $$Z_F = \sqrt{\mu/\epsilon} = \sqrt{\frac{\mu_0 \mu_r}{\epsilon_0 \epsilon_r}} = \sqrt{\frac{4\pi\times10^{-7}\times 1}{8.854\times10^{-12}\times 2}} = 266.39\,\Omega$$

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$$v_{ph} = \frac{c_0}{\sqrt{\mu_r \varepsilon_r}}$$

$\hat{e}_x$

(e) $$v_{ph} = \frac{c_0}{\sqrt{\mu_r \varepsilon_r}} = 2.12 \times 10^8\ \mathrm{ms^{-1}}$$

$$\vec{E} = E_0 \cos(\omega t + kz)\,\hat{e}_x$$

$$\vec{H} = H_0 \cos(\omega t + kz)\,(-\hat{e}_y)$$

(g) $E_0 = 1\ \mathrm{V/m}$

$$\vec{E} = \begin{pmatrix} 1\,\mathrm{V/m} \cdot e^{+jk_z z} \\ 0 \\ 0 \end{pmatrix}$$

e^{+} ...

$e^{0} = 1$

$$H_0 = \frac{E_0}{Z_F} = \frac{1}{266.39} = 3.75\ \mathrm{mA/m}$$

$\vec{H}^* = \ldots e^{-jk_z z}$

$$\vec{H} = \frac{1\ \mathrm{V/m}}{266.39}\, e^{+jk_z z} \begin{pmatrix} 0 \\ -1 \\ 0 \end{pmatrix}$$

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h) Compute the power flow through an area of 2.5 m^2 perpendicular to the direction of propagation.

$P = \left| \frac{1}{2} \vec{E} \times \vec{H}^* \right| \times 2.5$

$$\begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} \times \begin{pmatrix} -1 \\ 2 \\ 1 \end{pmatrix}$$

$$= \left| P_x \hat{e}_x + P_y \hat{e}_y \right|$$

$$\sqrt{P_x^2 + P_y^2}$$

(h) $P_{tot} = |\text{Poynting Vector}| \times \text{Area}$

$$= \left| \frac{1}{2} (\vec{E}^0 \times \vec{H}^{0*}) \right| \times 2.5\,\mathrm{m}^2$$

$$= \left| \frac{1}{2} \times 1 \times \frac{1}{266.39} \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \times \begin{pmatrix} 0 \\ -1 \\ 0 \end{pmatrix} \right| \times 2.5$$

$$= 4.69\ \mathrm{mW}$$

4.2 Problem 2

A lossless, homogeneous, linear and isotropic medium with a relative dielectric constant of $\varepsilon_r = 4$ and $\mu_r = 1$ is given. A harmonic and plane TEM wave with a wavelength of 30 mm and an amplitude of the electric field vector of $2\,\frac{\text{V}}{\text{m}}$ propagates in this medium in negative x-direction (cartesian coordinate system).

a) What can be concluded from the expression "harmonic and plane TEM wave"?

b) Compute the frequency f. ✓

c) Compute the wave vector $\vec{k}$. ✓

d) Compute the wave impedance Z_F. ✓

e) Compute the phase velocity v_{ph}. ✓

f) Formulate all electric and magnetic field components for a linear polarization in z-direction.

g) Compute the power flow through an area of $3\,\text{m}^2$, whose orientation is

i) $\vec{n}_A = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$ and

ii) $\vec{n}_A = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$

(c) $K = \frac{2\pi}{\lambda}$

$= \frac{2\pi}{30\times10^{-3}}$

$= 209.4395\ m^{-1}$

$\vec{K} = 209.4395\,m^{-1}\,(-\hat{e}_x)$

$= \begin{pmatrix} -209.43 \\ 0 \\ 0 \end{pmatrix}$

(b) $c = \frac{C_0}{\sqrt{\epsilon_r \mu_r}}$

$= 1.5\times10^{8}\ ms^{-1}$

$f = \frac{c}{\lambda}$

$= \frac{1.5\times10^{8}}{30\times10^{-3}}$

$= 5\times10^{9}\ Hz$

$= 5\ GHz$

(d) $Z_F = \sqrt{\mu/\epsilon} = 188.37\ \Omega$

(e) $V_{ph} = \frac{C_0}{\sqrt{\epsilon_r \mu_r}} = 1.5\times10^{8}\ ms^{-1}$

4.2 Problem 2

A lossless, homogeneous, linear and isotropic medium with a relative dielectric constant of $\varepsilon_r = 4$ and $\mu_r = 1$ is given. A harmonic and plane TEM wave with a wavelength of 30 mm and an amplitude of the electric field vector of $2\,\frac{\mathrm{V}}{\mathrm{m}}$ propagates in this medium in negative x-direction (cartesian coordinate system).

a) What can be concluded from the expression "harmonic and plane TEM wave"?

b) Compute the frequency f.

c) Compute the wave vector $\vec{k}$.

d) Compute the wave impedance Z_F.

e) Compute the phase velocity v_{ph}.

f) Formulate all electric and magnetic field components for a linear polarization in z-direction.

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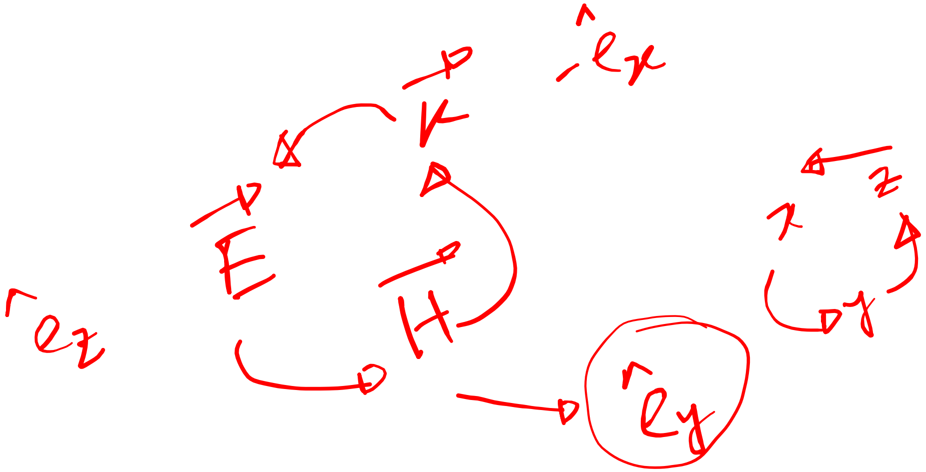


(f) $E_0 = 2\ \mathrm{V/m}$

$(\omega t - \vec{k}\cdot\vec{r})$

$-kx - ky$

$$\vec{E} = \begin{pmatrix} 0 \\ 0 \\ 2\,\mathrm{V/m} \cdot e^{jk_x x} \end{pmatrix}$$

$\vec{k} = k_x \hat{e}_x +$

$$H_0 = \frac{2}{188.37} = 0.0106\ \mathrm{A/m}$$

$\vec{k} = \begin{pmatrix} 2 \\ 3 \\ 4 \end{pmatrix}$

$(\omega t - 2x - 3y - 4z)$

$$\vec{H} = 0.0106 \cdot e^{jk_x x} \cdot \hat{e}_y = \begin{pmatrix} 0 \\ +0.0106\, e^{jk_x x} \\ 0 \end{pmatrix}$$

4.2 Problem 2

A lossless, homogeneous, linear and isotropic medium with a relative dielectric constant of $\varepsilon_r = 4$ and $\mu_r = 1$ is given. A harmonic and plane TEM wave with a wavelength of 30 mm and an amplitude of the electric field vector of $2\,\frac{\mathrm{V}}{\mathrm{m}}$ propagates in this medium in negative x-direction (cartesian coordinate system).

a) What can be concluded from the expression "harmonic and plane TEM wave"?

b) Compute the frequency f.

c) Compute the wave vector $\vec{k}$.

d) Compute the wave impedance Z_F.

e) Compute the phase velocity v_{ph}.

f) Formulate all electric and magnetic field components for a linear polarization in z-direction.

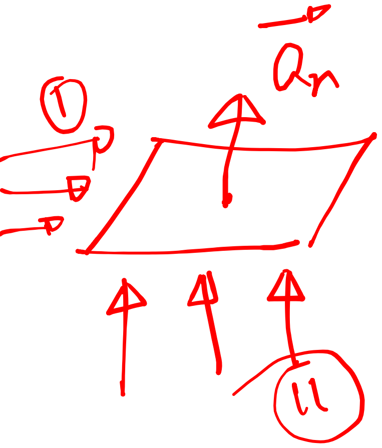
g) Compute the power flow through an area of $3\,\mathrm{m}^2$, whose orientation is

i) $\vec{n}_A = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$ and

ii) $\vec{n}_A = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$

$(\vec{a}_n \cdot \vec{P})$

$\vec{a}_n \perp \vec{P}$



(g) I $P = \frac{1}{2}(\vec{E} \times \vec{H}^*) \cdot \vec{A}$

$= \frac{1}{2}\begin{pmatrix} 0 \\ 0 \\ 2\,\mathrm{V/m}\, e^{jk_x x} \end{pmatrix} \times \begin{pmatrix} 0 \\ 0.0106\, e^{-jk_x x} \\ 0 \end{pmatrix} \cdot 3\,m^2 \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$

$\hat{e}_z \times \hat{e}_y$ $\quad -\hat{e}_x \cdot \hat{e}_y$

$= 0$

II $P = \frac{1}{2}(\vec{E} \times \vec{H}^*) \cdot \vec{A}$

$= \frac{1}{2}\begin{pmatrix} 0 \\ 0 \\ 2\,\mathrm{V/m}\, e^{jk_x x} \end{pmatrix} \times \begin{pmatrix} 0 \\ 0.0106\, e^{-jk_x x} \\ 0 \end{pmatrix} \cdot 3\,m^2 \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$

$= 31.8\ \mathrm{mW}$

$\hat{e}_z \times \hat{e}_y$ $\quad -\hat{e}_x \cdot \hat{e}_x$

4.3 Problem 3

The electric field strength of a harmonic and plane TEM wave with a frequency of 24 GHz is given as:

$$\vec{E} = 4\frac{\text{V}}{\text{m}}\cos(\omega t - \frac{\pi}{2} + 800\frac{1}{\text{m}}z)\cdot\vec{e}_x + 2\frac{\text{V}}{\text{m}}\cos(\omega t + 800\frac{1}{\text{m}}z)\cdot\vec{e}_y.$$

The lossless, homogeneous and isotropic medium has a relative magnetic permeability of $\mu_r = 1$.

a) What is the polarization of this TEM wave and what is the direction of propagation? Explain.

b) Give the wave number and the wave vector.

c) Determine the relative dielectric constant ε_r of the medium.

d) Determine the characteristic wave impedance Z_F.

e) Determine the wavelength λ.

f) Give the magnetic vector field $\vec{H}$.

(a) RHEP, negative z direction

(b) $k = 800\,m^{-1}$, $\vec{k} = \begin{pmatrix} 0 \\ 0 \\ -800 \end{pmatrix} = 800^{-m}(-\hat{e}_z)$

(c) $k = \omega\sqrt{\mu\epsilon}$

$k^2 = \omega^2 \mu_r \mu_0 \epsilon_r \epsilon_0$

$\epsilon_r = \dfrac{k^2}{\omega^2 \mu_r \mu_0 \times \epsilon_0}$

$= \dfrac{800^2}{(2\pi \times 24 \times 10^9)^2 \times 4\pi \times 10^{-7} \times 8.854 \times 10^{-12}}$

$= 2.5$

4.3 Problem 3

The electric field strength of a harmonic and plane TEM wave with a frequency of 24 GHz is given as:

$$\vec{E} = 4\frac{\text{V}}{\text{m}}\cos(\omega t - \frac{\pi}{2} + 800\frac{1}{\text{m}}z)\cdot\vec{e}_x + 2\frac{\text{V}}{\text{m}}\cos(\omega t + 800\frac{1}{\text{m}}z)\cdot\vec{e}_y.$$

The lossless, homogeneous and isotropic medium has a relative magnetic permeability of $\mu_r = 1$.

a) What is the polarization of this TEM wave and what is the direction of propagation? Explain.

b) Give the wave number and the wave vector.

c) Determine the relative dielectric constant ε_r of the medium.

d) Determine the characteristic wave impedance Z_F.

e) Determine the wavelength λ.

f) Give the magnetic vector field $\vec{H}$.

(d) $Z_F = \sqrt{\frac{\mu}{\varepsilon}}$

$= \sqrt{\frac{\mu_r \mu_0}{\varepsilon_r \varepsilon_0}}$

$= \sqrt{\frac{1 \times 4\pi \times 10^{-7}}{2.5 \times 8.854 \times 10^{-12}}}$

$= 238.27\,\Omega$

(e) $\lambda_g = \frac{2\pi}{K}$

$= \frac{2\pi}{800}$

$= 7.85 \times 10^{-3}\,\text{m} = 7.85\,\text{mm}$

4.3 Problem 3

The electric field strength of a harmonic and plane TEM wave with a frequency of 24 GHz is given as:

$$\vec{E} = 4\frac{\text{V}}{\text{m}}\cos(\omega t - \frac{\pi}{2} + 800\frac{1}{\text{m}}z)\cdot\vec{e}_x + 2\frac{\text{V}}{\text{m}}\cos(\omega t + 800\frac{1}{\text{m}}z)\cdot\vec{e}_y.$$

The lossless, homogeneous and isotropic medium has a relative magnetic permeability of $\mu_r = 1$.

a) What is the polarization of this TEM wave and what is the direction of propagation? Explain.

b) Give the wave number and the wave vector.

c) Determine the relative dielectric constant ε_r of the medium.

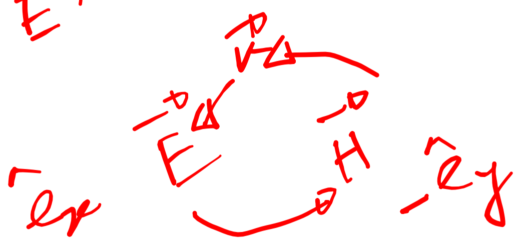
d) Determine the characteristic wave impedance Z_F.

e) Determine the wavelength λ.

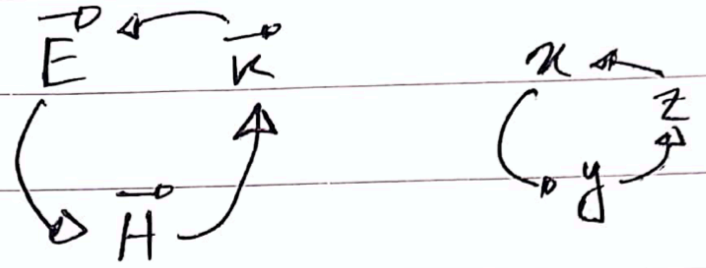
f) Give the magnetic vector field $\vec{H}$.

$$\nabla \times \vec{E} = -\mu_0 \frac{\partial \vec{H}}{\partial t}$$

$$\vec{E} \times \vec{H} = \vec{K} \quad \hat{e}_x \quad -\hat{e}_z$$



4.3 (f)



$$\vec{H} = (\text{Poynting Vector}) \times (\vec{E})$$

$$\vec{H} = \frac{1}{Z_F}\begin{pmatrix} 2\,\text{V/m}\cos(\omega t + 800z) \\ -4\,\text{V/m}\cos(\omega t - \pi/2 + 800z) \\ 0 \end{pmatrix} \quad \begin{array}{l} -\hat{e}_z \times \hat{e}_x = -\hat{e}_y \\ -\hat{e}_z \times \hat{e}_y = \hat{e}_x \end{array}$$

4.4 Problem 4

A harmonic and plane TEM wave with a frequency of 2 GHz propagates in positive x-direction (Cartesian coordinate system). The magnetic field has a maximum amplitude of 0.029 $\frac{\text{A}}{\text{m}}$ in an arbitrary point P. The lossless, homogeneous and isotropic medium has a relative dielectric constant of $\varepsilon_r = 4.5$ and a relative magnetic permeability of $\mu_r = 1$. The wave is linearly polarized along the z-axis.

a) Determine the wave number k and the wave vector $\vec{k}$.

b) Determine the wavelength λ.

c) Compute the wave impedance Z_F.

d) Determine the amplitude of the electric field strength in point P.

e) Formulate the electric and the magnetic vector field components of the wave.

f) Extend the expression of the electric vector field to give a left-handed circularly polarized wave.

(a) $k = \omega\sqrt{\mu\epsilon}$

$= 2\pi \times 2 \times 10^9 \times \sqrt{4\pi \times 10^{-7} \times 4.5 \times 8.854 \times 10^{-12}}$

$= 88.92$

$$\vec{k} = \begin{pmatrix} 88.92 \\ 0 \\ 0 \end{pmatrix} = 88.92\,(+\hat{e}_x)$$

(b) $$V_{ph} = \frac{c_0}{\sqrt{\epsilon_r \mu_r}} = \frac{3 \times 10^8}{\sqrt{4.5 \times 1}} = 1.41 \times 10^8\ ms^{-1}$$

$$\lambda = \frac{V_{ph}}{f} = 70.71\ mm$$

4.4 Problem 4

A harmonic and plane TEM wave with a frequency of 2 GHz propagates in positive x-direction (Cartesian coordinate system). The magnetic field has a maximum amplitude of 0.029 $\frac{\mathrm{A}}{\mathrm{m}}$ in an arbitrary point P. The lossless, homogeneous and isotropic medium has a relative dielectric constant of $\varepsilon_r = 4.5$ and a relative magnetic permeability of $\mu_r = 1$. The wave is linearly polarized along the z-axis.

a) Determine the wave number k and the wave vector $\vec{k}$.

b) Determine the wavelength λ.

c) Compute the wave impedance Z_F.

d) Determine the amplitude of the electric field strength in point P.

e) Formulate the electric and the magnetic vector field components of the wave.

f) Extend the expression of the electric vector field to give a left-handed circularly polarized wave.

$Q \sim 0.09$

c) $Z_F = \sqrt{\mu / \epsilon}$

$= 177.59\ \Omega$

$E_p = E_0 \cos(\omega t - kx)$

d) $E_p = H_p \times Z_F$

$= 5.15011\ \mathrm{V/m}$

4.4 Problem 4

A harmonic and plane TEM wave with a frequency of 2 GHz propagates in positive x-direction (Cartesian coordinate system). The magnetic field has a maximum amplitude of 0.029 $\frac{\text{A}}{\text{m}}$ in an arbitrary point P. The lossless, homogeneous and isotropic medium has a relative dielectric constant of $\varepsilon_r = 4.5$ and a relative magnetic permeability of $\mu_r = 1$. The wave is linearly polarized along the z-axis.

a) Determine the wave number k and the wave vector $\vec{k}$.

b) Determine the wavelength λ.

c) Compute the wave impedance Z_F.

d) Determine the amplitude of the electric field strength in point P.

e) Formulate the electric and the magnetic vector field components of the wave.

f) Extend the expression of the electric vector field to give a left-handed circularly polarized wave.

$$E_y = 5\cos(\omega t - k_x x + \pi/2)$$

$$E_z = 5\cos(\omega t - k_x x)$$

(e)

$$\vec{E} = 5\,V/m \cdot e^{j\omega t} \cdot e^{-jk_x x}\begin{pmatrix}0\\0\\1\end{pmatrix}$$

At a random time (t)

$$\vec{E} \times \vec{H} = \vec{k}$$

$$\hat{e}_z \times (-\hat{e}_y) = \hat{e}_x$$

$$\vec{H} = 0.029\,A/m \cdot e^{j\omega t} \cdot e^{-jk_x x}\begin{pmatrix}0\\-1\\0\end{pmatrix}$$

(f)

$$\vec{E} = 5\,V/m \cdot e^{j\omega t} \cdot e^{-jk_x x}\begin{pmatrix}0\\e^{+j\pi/2}\\1\end{pmatrix}$$

For LHCP, $\phi = \pi/2$

$$\vec{E} = E_y\hat{e}_y + E_z\hat{e}_z$$

4.5 Problem 5

A plane and harmonic TEM wave propagates into the negative x-direction in a lossless dielectric with a relative permittivity of $\varepsilon_r = 4$. The wave is linearly polarized along the line $z = 2y$. The wave number is $k = 418.88\,\frac{1}{\text{m}}$. The magnitude of the magnetic vector field is $H_0 = 5.3\,\frac{\text{mA}}{\text{m}}$.

a) Calculate the frequency f of the wave.

b) Calculate the characteristic wave impedance.

c) Determine the magnitude E_0 of the electric field strength.

d) Formulate the vector fields of the electric and magnetic field strengths of the wave in time domain.

e) Determine the Poynting vector $\vec{S}(t)$ of the wave.

f) Determine the averaged power density $\overline{\vec{S}}$ over one time period T.

a) $$v_{ph} = \frac{c_0}{\sqrt{\epsilon_r \mu_r}}$$

$$= 1.5 \times 10^8\, ms^{-1}$$

$$k = \frac{2\pi}{\lambda}$$

$$\Rightarrow \lambda = \frac{2\pi}{k} = 0.015\, m$$

$$f = \frac{v_{ph}}{\lambda}$$

$$= 10^{10}\, Hz$$

$$= 10\, GHz$$

4.5 Problem 5

A plane and harmonic TEM wave propagates into the negative x-direction in a lossless dielectric with a relative permittivity of $\varepsilon_r = 4$. The wave is linearly polarized along the line $z = 2y$. The wave number is $k = 418.88\,\frac{1}{\mathrm{m}}$. The magnitude of the magnetic vector field is $H_0 = 5.3\,\frac{\mathrm{mA}}{\mathrm{m}}$.

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d) Formulate the vector fields of the electric and magnetic field strengths of the wave in time domain.

e) Determine the Poynting vector $\vec{S}(t)$ of the wave.

f) Determine the averaged power density $\overline{\vec{S}}$ over one time period T.

b

$$Z_F = \sqrt{\frac{\mu}{\epsilon}}$$

$$= \sqrt{\frac{\mu_r \mu_0}{\epsilon_r \epsilon_0}}$$

$$= 188.4\,\Omega$$

4.5 Problem 5

A plane and harmonic TEM wave propagates into the negative x-direction in a lossless dielectric with a relative permittivity of $\varepsilon_r = 4$. The wave is linearly polarized along the line $z = 2y$. The wave number is $k = 418.88\,\frac{1}{\text{m}}$. The magnitude of the magnetic vector field is $H_0 = 5.3\,\frac{\text{mA}}{\text{m}}$.

a) Calculate the frequency f of the wave.

b) Calculate the characteristic wave impedance.

c) Determine the magnitude E_0 of the electric field strength.

d) Formulate the vector fields of the electric and magnetic field strengths of the wave in time domain.

e) Determine the Poynting vector $\vec{S}(t)$ of the wave.

f) Determine the averaged power density $\overline{\vec{S}}$ over one time period T.

c) $E_0 = H_0 \times Z_F$

$= 5.3 \times 10^{-3} \times 188.4\ \text{V/m}$

$= \underline{1}\ \text{V/m}$

4.5 Problem 5

A plane and harmonic TEM wave propagates into the negative x-direction in a lossless dielectric with a relative permittivity of $\varepsilon_r = 4$. The wave is linearly polarized along the line $z = 2y$. The wave number is $k = 418.88\,\frac{1}{\text{m}}$. The magnitude of the magnetic vector field is $H_0 = 5.3\,\frac{\text{mA}}{\text{m}}$.

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d) Formulate the vector fields of the electric and magnetic field strengths of the wave in time domain.

e) Determine the Poynting vector $\vec{S}(t)$ of the wave.

f) Determine the averaged power density $\overline{\vec{S}}$ over one time period T.

$$\hat{a}_n = \frac{\nabla\phi}{|\nabla\phi|} \quad \hat{e}_x$$

along the line

x axis

$x = \ldots$

$\hat{e}_x$

line vector

$$\vec{H} = |\vec{H}|\,(\hat{a}_n)$$

$$= H_0 \cos(\omega t + k_x x) \cdot \frac{2\hat{e}_y - \hat{e}_z}{\sqrt{5}} \quad \text{w}$$

Wave direction at negative x

$$= \frac{H_0}{\sqrt{5}} \cos(\omega t + k_x x) \begin{pmatrix} 0 \\ 2 \\ -1 \end{pmatrix} \quad \text{w}$$

$$\boxed{\begin{aligned} \vec{H} &= H_0 \cos(\omega t + k_x x)\,(+\hat{e}_y) \\ \vec{E} &= E_0 \cos(\omega t + k_x x)\,(\hat{e}_z) \end{aligned}}$$

Not included

Normal unit vector along the line, $2y - z = 0$ is:

$$\hat{a}_n = \frac{\nabla\phi}{|\nabla\phi|} = \frac{2\hat{e}_y - \hat{e}_z}{\sqrt{5}}$$

x, z, y

$\vec{E} \times \vec{H}$ must be $(-\hat{e}_x)$

$$\therefore \vec{E} = \frac{E_0}{\sqrt{5}} \cos(\omega t + k_x x) \begin{pmatrix} 0 \\ +1 \\ +2 \end{pmatrix}$$

$\vec{E}$, $\vec{k}\,(-\vec{e}_x)$, $\vec{H}$

$\pm\vec{e}_z$

$$\vec{e}_y \times (-\hat{e}_x)$$

$$\hat{e}_x \times \hat{e}_y = \hat{e}_z$$

4.5 Problem 5

A plane and harmonic TEM wave propagates into the negative x-direction in a lossless dielectric with a relative permittivity of $\varepsilon_r = 4$. The wave is linearly polarized along the line $z = 2y$. The wave number is $k = 418.88\,\frac{1}{\text{m}}$. The magnitude of the magnetic vector field is $H_0 = 5.3\,\frac{\text{mA}}{\text{m}}$.

a) Calculate the frequency f of the wave.

b) Calculate the characteristic wave impedance.

c) Determine the magnitude E_0 of the electric field strength.

d) Formulate the vector fields of the electric and magnetic field strengths of the wave in time domain.

e) Determine the Poynting vector $\vec{S}(t)$ of the wave.

f) Determine the averaged power density $\overline{\vec{S}}$ over one time period T.

$\frac{1}{2}\vec{E}\times\vec{H}^*$

$2y - z = 0$

$\vec{H}$ linearly polarized along $z = 2y$

$\vec{\nabla}\phi$

$\hat{a}_{line} = \frac{\vec{\nabla}\phi}{|\vec{\nabla}\phi|} = \frac{2\hat{e}_y - \hat{e}_z}{\sqrt{5}}$

(e) Poynting vector,

$\frac{1}{\sqrt{5}}\begin{pmatrix}0\\1\\2\end{pmatrix} \times \frac{1}{\sqrt{5}}\begin{pmatrix}0\\2\\-1\end{pmatrix}$

$$\vec{S}(t) = \vec{E}(t)\times\vec{H}(t) \quad W$$

$$= \frac{E_0 H_0}{5}\cos^2(\omega t + k_x x)\left[\begin{pmatrix}0\\2\\1\end{pmatrix}\times\begin{pmatrix}0\\1\\-2\end{pmatrix}\right]$$

$$= E_0 H_0 \cos^2(\omega t + k_x x)\,(-\hat{e}_x)$$

$$= 1\times 5.3\times 10^{-3}\cos^2(\omega t + 418.88\,x)(-\hat{e}_x)\ \text{W/m}^2$$

4.5 Problem 5

A plane and harmonic TEM wave propagates into the negative x-direction in a lossless dielectric with a relative permittivity of $\varepsilon_r = 4$. The wave is linearly polarized along the line $z = 2y$. The wave number is $k = 418.88\,\frac{1}{\text{m}}$. The magnitude of the magnetic vector field is $H_0 = 5.3\,\frac{\text{mA}}{\text{m}}$.

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d) Formulate the vector fields of the electric and magnetic field strengths of the wave in time domain.

e) Determine the Poynting vector $\vec{S}(t)$ of the wave.

f) Determine the averaged power density $\overline{\vec{S}}$ over one time period T.

(f) $\vec{S} = \frac{1}{2} \mathbb{R}\{\vec{E} \times \vec{H}^*\}$

$= 1/T \int \vec{S}(t)\,dt$

$\vec{S} = -1/2 \frac{E_0^2}{Z_F} \hat{e}_x$

$= -1/2 \times \frac{1^2}{188.4} \hat{e}_x$

$= -2.65 \times 10^{-3} \hat{e}_x$