Lecture 8

Polarization

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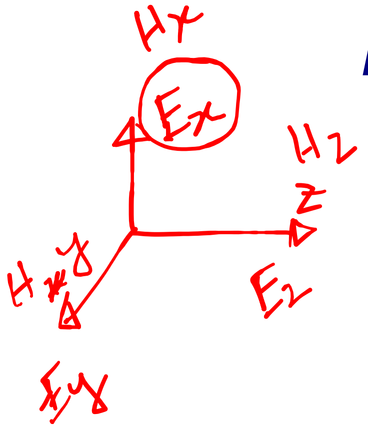
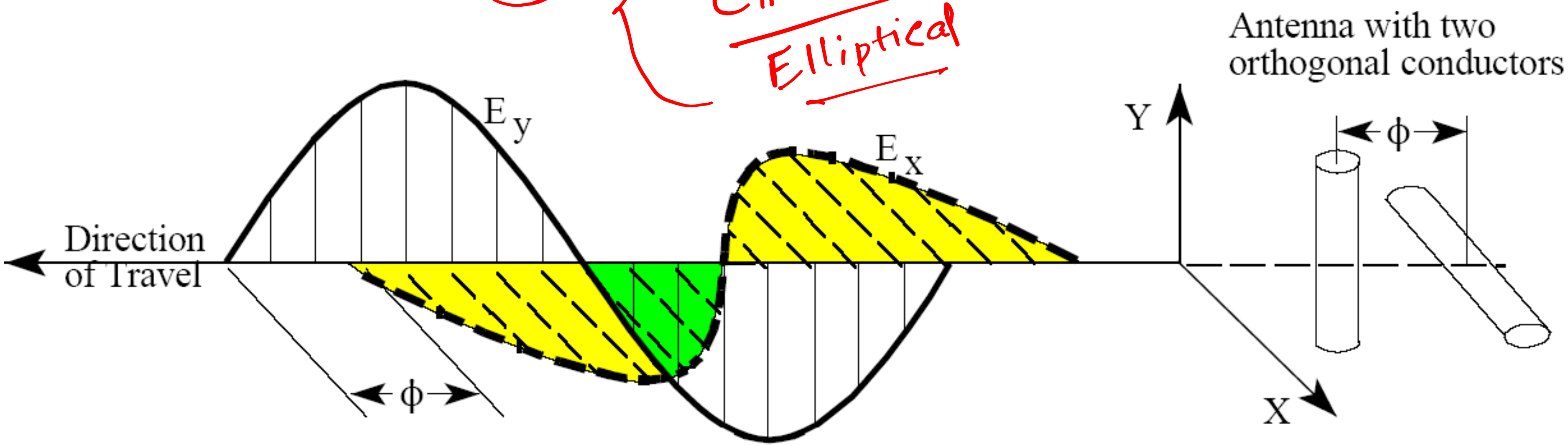
Polarization of EM Waves



Linear
Circular
Elliptical

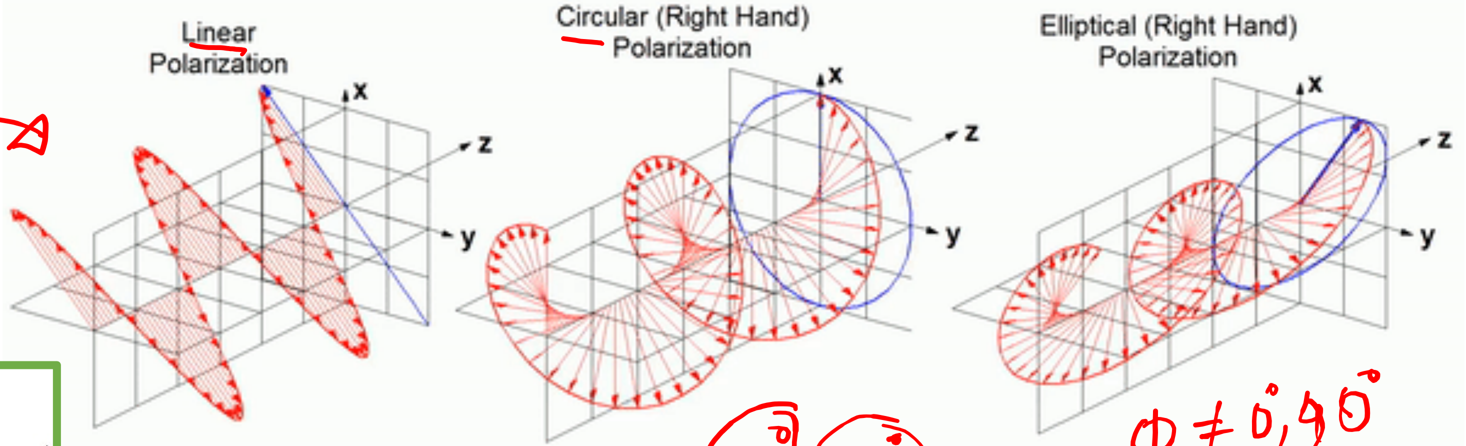


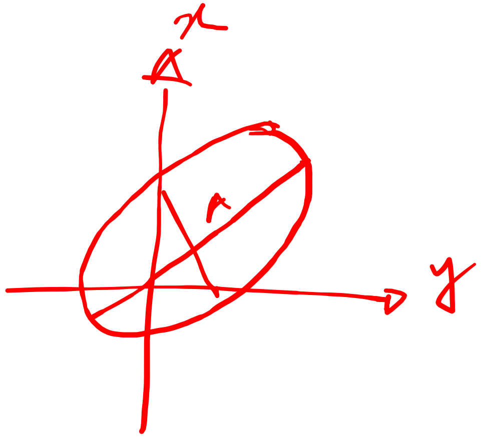
The sum of the E field vectors determines the sense of polarization

Polarization is the property of wave that can **oscillate with more than one orientation**. A light wave that is vibrating in more than one plane is referred to as unpolarized light. The process of transforming unpolarized light into polarized light is known as **polarization of light**.

Polarization

RHCP Clockwise





$$\left\{\begin{array}{lcl} E_x & = & A\cos(\omega t - k_z z) \\ E_y & = & B\cos(\omega t - k_z z + \varphi) \end{array}\right.$$

$\phi = 0^\circ, 180^\circ$

$\phi = 90^\circ, 270^\circ$

$\phi \neq 0^\circ, 90^\circ$

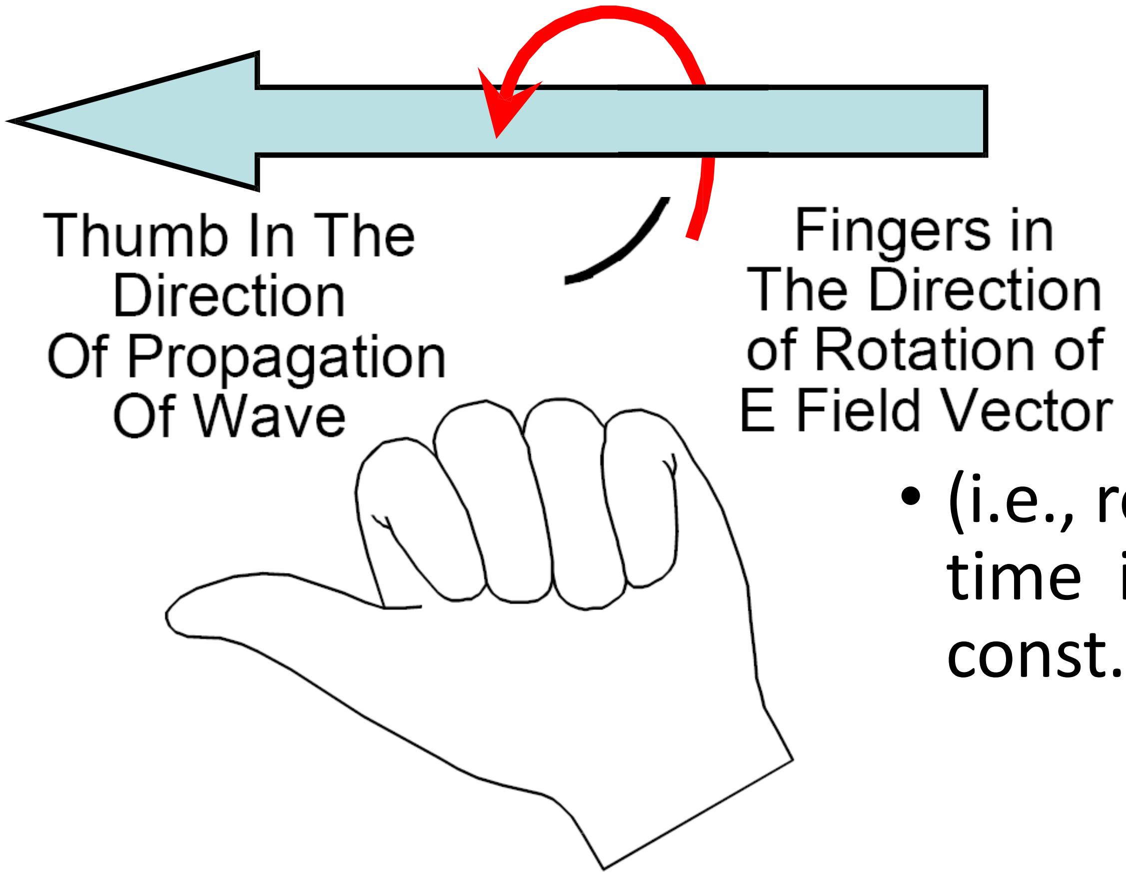
$A > B$

Linear polarization: When an ordinary (unpolarized) light is reflected from a polished surface or transmission through certain materials, the electric fields vector oscillates **along a straight line** in one plane, and the light is said to be linearly polarized.

Circular polarization: The electric field of light consists of two linear components that are perpendicular to each other, equal in amplitude, but **have a phase difference of π/2**. The resulting electric field **rotates in a circle** around the direction of propagation and, depending on the rotation direction, is called left- or right-hand circularly polarized light.

Elliptical polarization: The **electric field of light describes an ellipse.** This results from the combination of two linear components with differing amplitudes or a **phase difference that is not π/2.**

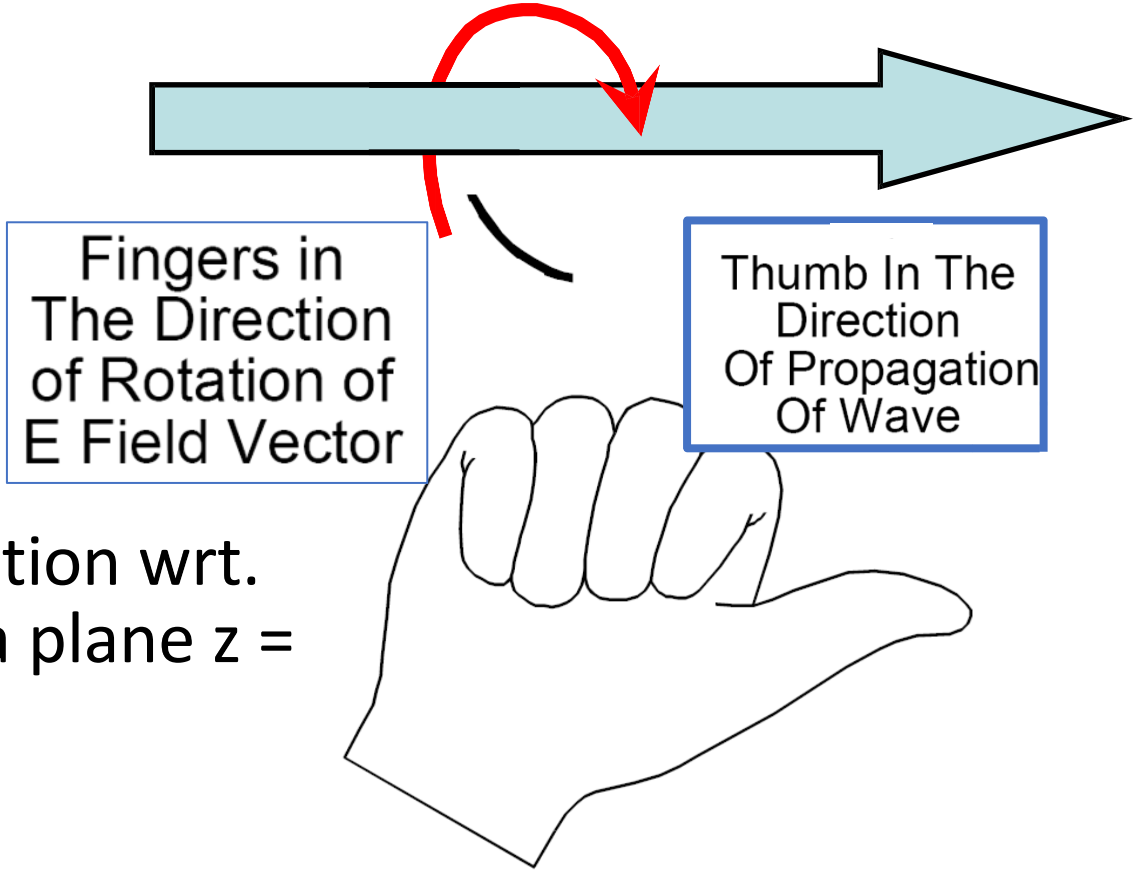
Polarization of EM Waves



- (i.e., rotation wrt. time in a plane z = const.)

LEFT HAND POLARIZATION

Polarization of EM Waves



- (i.e., rotation wrt. time in a plane z = const.)

RIGHT HAND POLARIZATION

Polarization of EM Waves

TE
TM
TEM

$E_z = 0, H_z = 0$

**TEM wave propagating harmonically in the z - direction

$$E_x = A\cos(\omega t - k_z z)$$
$$E_y = B\cos(\omega t - k_z z + \varphi)$$

$$u = \omega t - k_z z_0 + \frac{\varphi}{2}$$

$$E_x/A = \cos(u - \varphi/2) = \cos u \cos\frac{\varphi}{2} + \sin u \sin\frac{\varphi}{2} \quad \text{— (I)}$$
$$E_y/B = \cos(u + \varphi/2) = \cos u \cos\frac{\varphi}{2} - \sin u \sin\frac{\varphi}{2} \quad \text{— (II)}$$

Adding and subtracting both equations yields (I) + (II)

$$E_x/A + E_y/B = 2\cos u \cos\frac{\varphi}{2} \quad \text{— (III)}$$
$$E_x/A - E_y/B = 2\sin u \sin\frac{\varphi}{2} \quad \text{— (IV)}$$

Now we divide the first (III) equation by $2\cos\frac{\varphi}{2}$ and the second (IV) by $2\sin\frac{\varphi}{2}$:

$$\frac{E_x}{2A\cos\frac{\varphi}{2}} + \frac{E_y}{2B\cos\frac{\varphi}{2}} = \cos u$$
$$\frac{E_x}{2A\sin\frac{\varphi}{2}} - \frac{E_y}{2B\sin\frac{\varphi}{2}} = \sin u$$

$$\boxed{\cos^2 u + \sin^2 u = 1}$$

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

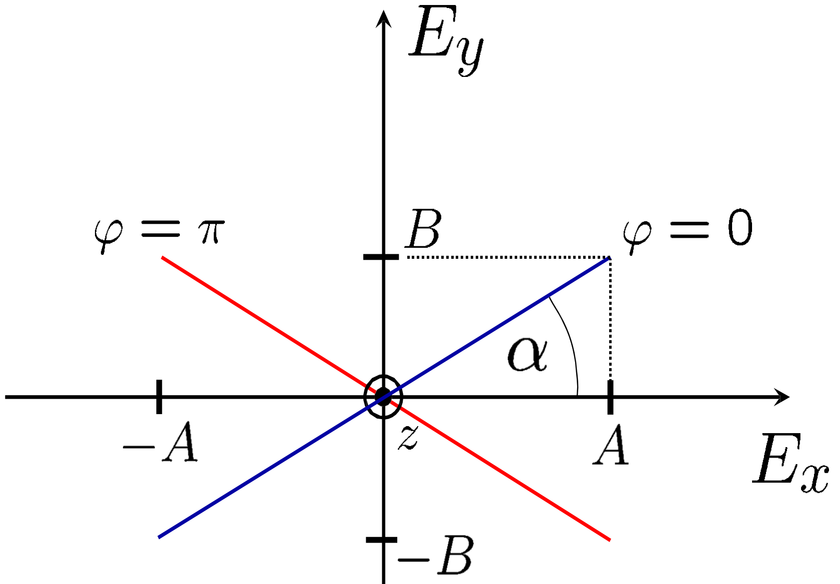
Ellipse

$$\left(\frac{E_x}{2A\cos\frac{\varphi}{2}} + \frac{E_y}{2B\cos\frac{\varphi}{2}}\right)^2 + \left(\frac{E_x}{2A\sin\frac{\varphi}{2}} - \frac{E_y}{2B\sin\frac{\varphi}{2}}\right)^2 = 1$$

***Equation of an elliptically polarized plane TEM wave

Linear Polarization

$\phi = 2\pi$



$\tan\alpha = (-1)^m \frac{B}{A}$

$$\boxed{\begin{aligned} E_x &= A\cos(\omega t - k_z z) \\ E_y &= B\cos(\omega t - k_z z + \varphi) \end{aligned}}$$

$$\boxed{\begin{aligned} E_x/A + E_y/B &= 2\cos u \cancel{\cos\frac{\varphi}{2}} \to 1 \quad \text{--- (i)} \\ E_x/A - E_y/B &= 2\sin u \cancel{\sin\frac{\varphi}{2}} \to 0 \quad \text{--- (ii)} \end{aligned}}$$

1.) $\varphi = \pm m\pi$, $m = 0, 1, 2, \ldots \Rightarrow$ **linearly polarized wave, linear polarization**
The direction of $\vec{E}$ (i.e., the $\vec{E}$ plane) is fixed.

1a) $m = 0 \Rightarrow \varphi = 0$, $\sin\frac{\varphi}{2} = 0$, $\cos\frac{\varphi}{2} = 1$

$0, 2\pi, 4\pi \ldots \rightarrow$

Blue line

$E_x/A - E_y/B = 0$ [From (ii)]

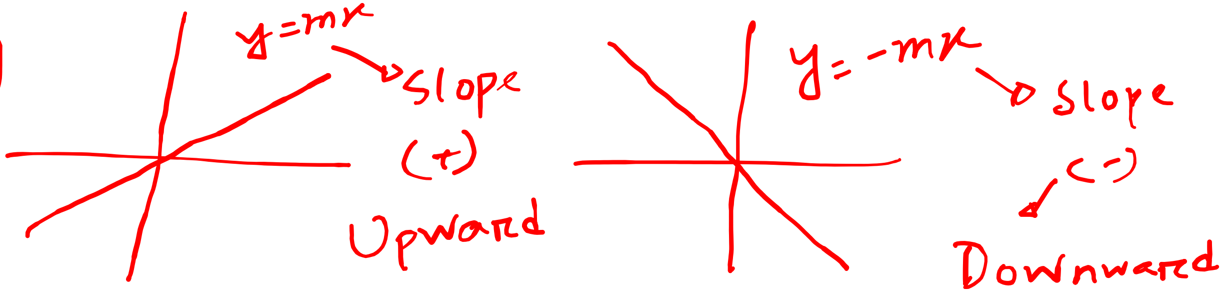
1b) $m = 1 \Rightarrow \varphi = \pi$, $\sin\frac{\varphi}{2} = 1$, $\cos\frac{\varphi}{2} = 0$

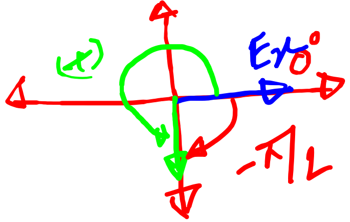
$\pi, 3\pi, 5\pi \ldots \rightarrow$

red line

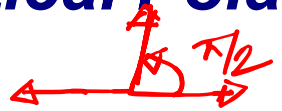
$E_x/A + E_y/B = 0$ [From (i)]

$y = mx$





Elliptical Polarization



$$\left(\frac{E_x}{2A\cos\frac{\varphi}{2}}+\frac{E_y}{2B\cos\frac{\varphi}{2}}\right)^2+\left(\frac{E_x}{2A\sin\frac{\varphi}{2}}-\frac{E_y}{2B\sin\frac{\varphi}{2}}\right)^2=1 \quad \text{— (i)}$$

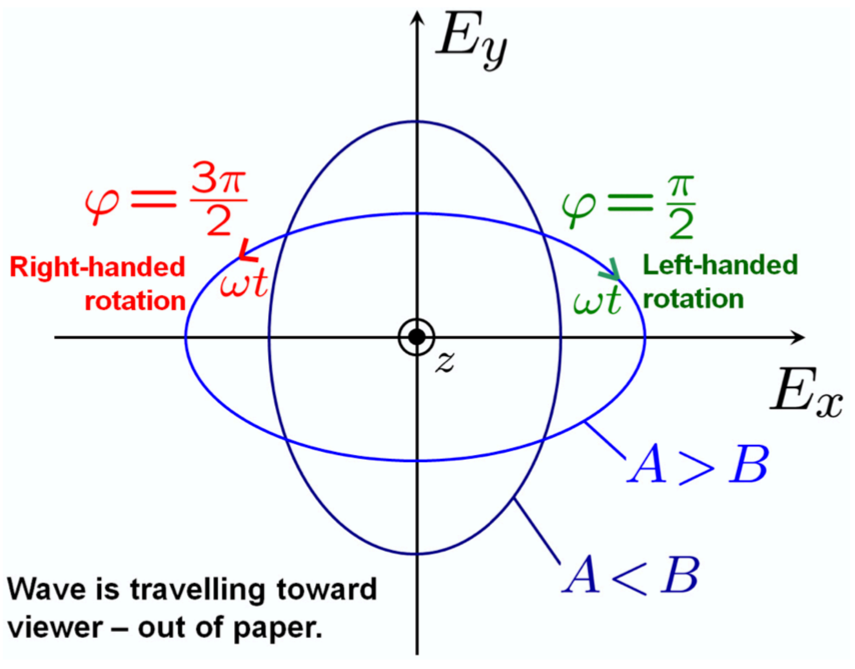


Figure 4.3: Elliptical polarization

2.) $\varphi = m\frac{\pi}{2}$, $m = \pm1, \pm3, \ldots \Rightarrow$ **elliptical polarization with the ellipse's main axes in x- and y-direction** (see Fig. 4.3).

45° $\phi = \pi/2$

π/2. হলে ওপরে দিক

$\boxed{A \neq B}$ *

$$\sin^2\frac{\varphi}{2} = \cos^2\frac{\varphi}{2} = \frac{1}{2}$$

$$\left(\frac{E_x}{A}+\frac{E_y}{B}\right)^2+\left(\frac{E_x}{A}-\frac{E_y}{B}\right)^2 = 2$$ [(i) এ value plugin করে]

$$\left(\frac{E_x}{A}\right)^2+\left(\frac{E_y}{B}\right)^2 = 1$$

$x^2/a^2 + y^2/b^2 = 1$

$$\begin{aligned} E_x &= A\cos(\omega t - k_z z) \\ E_y &= B\cos(\omega t - k_z z + \varphi) \end{aligned}$$

left-hand circular polarization (LHCP): $\varphi = \frac{\pi}{2}$

right-hand circular polarization (RHCP): $\varphi = \frac{3\pi}{2} = -\frac{\pi}{2}$

Circular Polarization

A=B

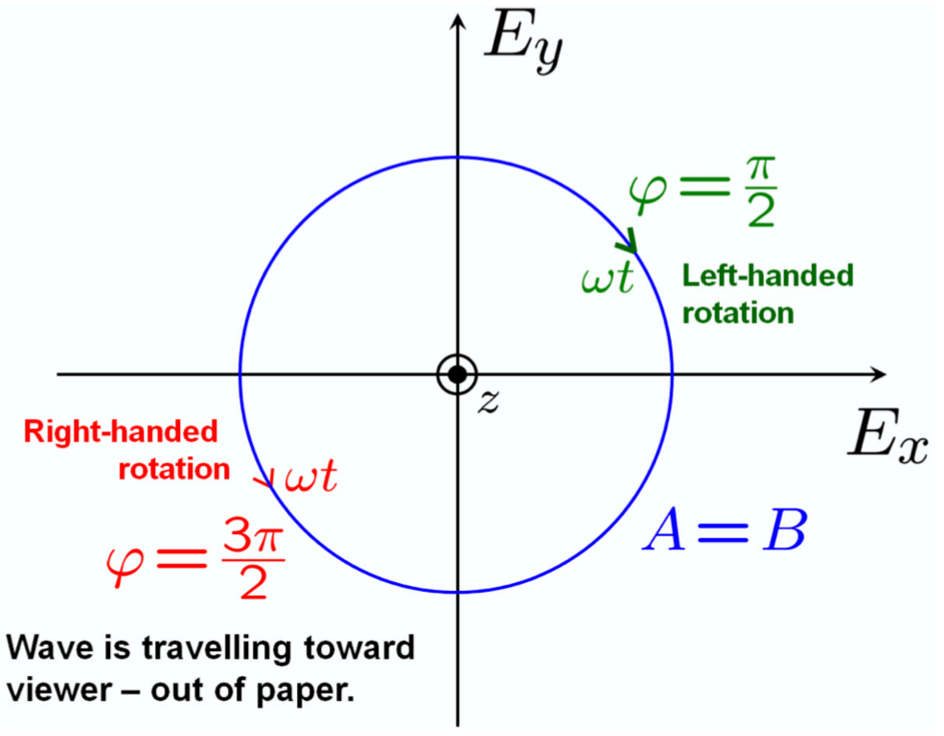


Figure 4.4: Circular polarization

$$E_x = A\cos(\omega t - k_z z)$$
$$E_y = B\cos(\omega t - k_z z + \varphi)$$

$$\sin^2\frac{\varphi}{2} = \cos^2\frac{\varphi}{2} = \frac{1}{2}$$
$$\left(\frac{E_x}{A} + \frac{E_y}{B}\right)^2 + \left(\frac{E_x}{A} - \frac{E_y}{B}\right)^2 = 2$$

$x^2 + y^2 = R^2$

$$\left(\frac{E_x}{A}\right)^2 + \left(\frac{E_y}{B}\right)^2 = 1$$

3.) $\varphi = m\frac{\pi}{2}$, $m = \pm1, \pm3, \ldots$ and $A = B \Rightarrow$ **circular polarization**

$$E_x^2 + E_y^2 = A^2$$

left-hand circular polarization (LHCP): $\varphi = \frac{\pi}{2}$

right-hand circular polarization (RHCP): $\varphi = \frac{3\pi}{2} = -\frac{\pi}{2}$

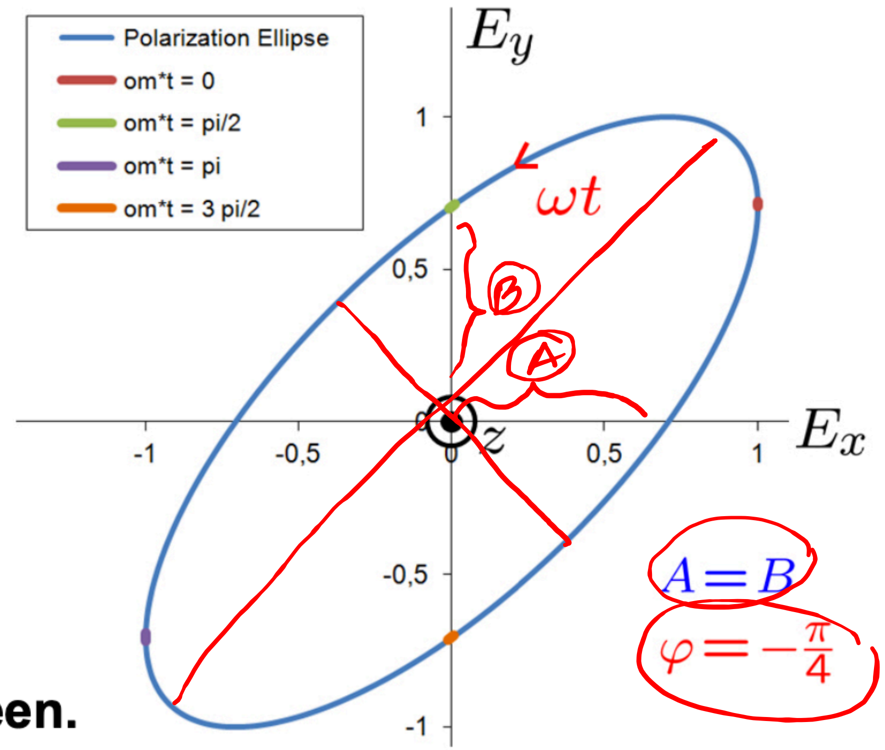
Elliptical Polarization (RHEP)

$\phi = 90°$

$A \neq B$

$\phi = 0°, 90°$

RHEP – Right-hand elliptical



$$\boxed{\begin{aligned} E_x &= A\cos(\omega t - k_z z) \\ E_y &= B\cos(\omega t - k_z z + \cancel{\varphi}) \end{aligned}}$$

$-\pi/4$

$A = B$

$\varphi = -\frac{\pi}{4}$

Wave is travelling toward viewer – out of paper/wall/screen.

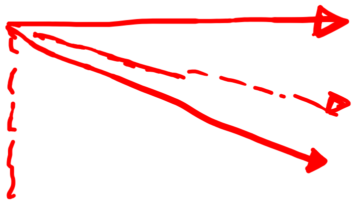
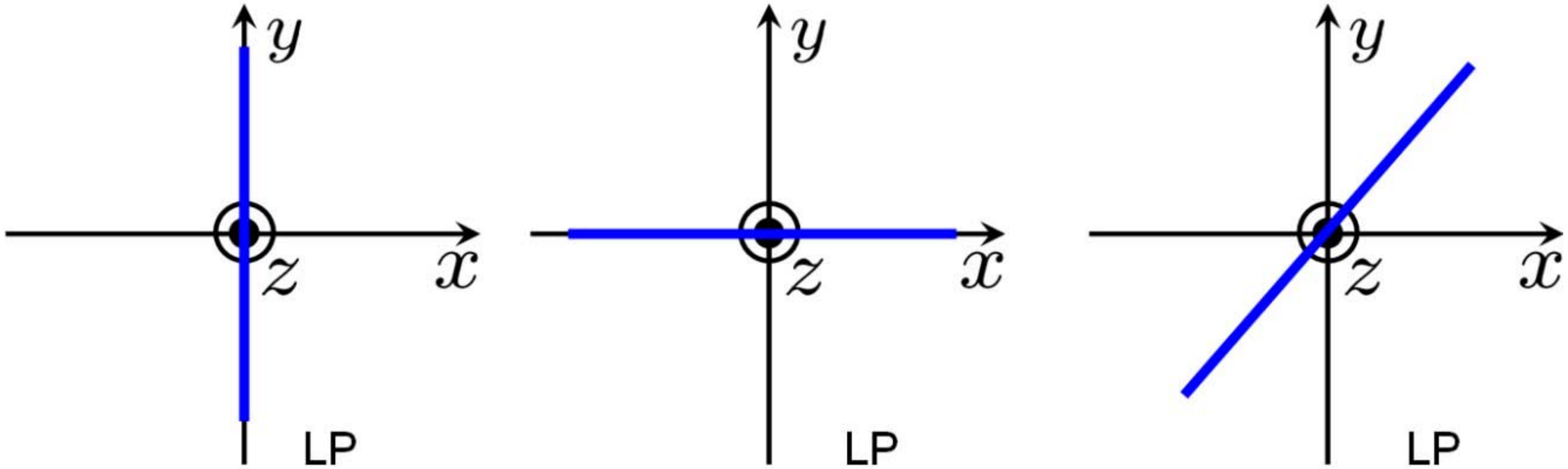


Figure 4.5: Elliptical polarization (RHEP) with $A = B$ and $\varphi = -45°$

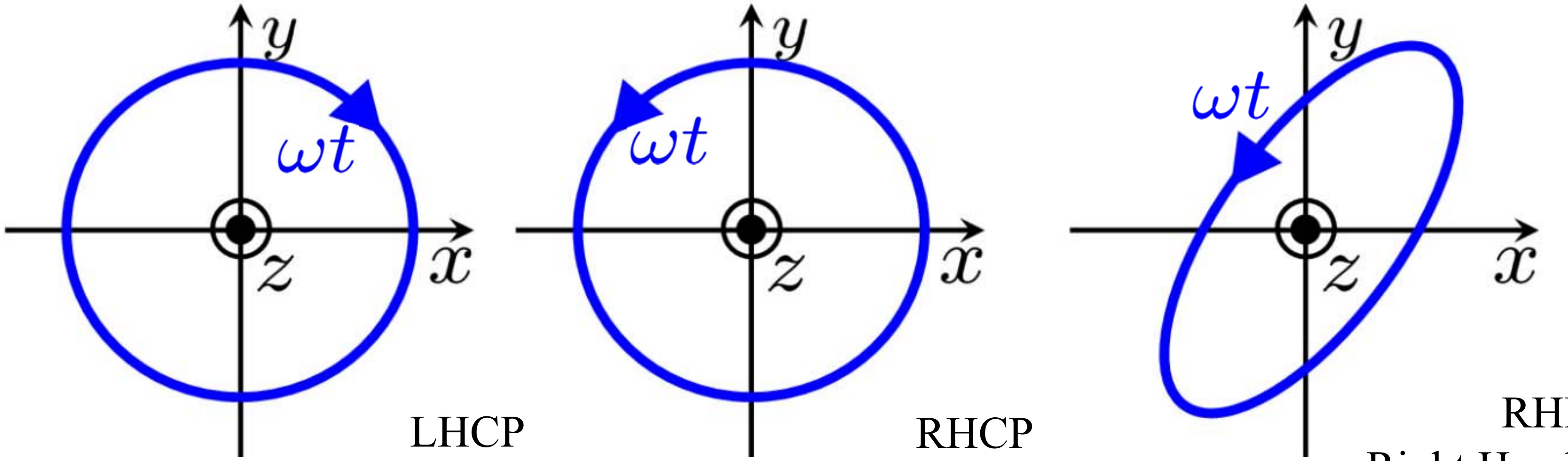
Please note that even if both magnitudes are equal ($A = B$), the polarization becomes elliptical if angle φ is not equal $\pm 90°$. Fig. 4.5 shows this clearly for a phase shift of $\varphi = -45°$ and $A = B$, the polarization is right-hand elliptical (RHEP).

Linear, Elliptical, and Circular Polarization



Wave is travelling toward viewer – out of paper.

Check Test!!!



LHCP
Left Hand Circular Polarization

RHCP
Right Hand Circular Polarization

RHEP
Right Hand Elliptical Polarization

Linear, Elliptical, and Circular Polarization

$0 \leftarrow$ linear (LP)

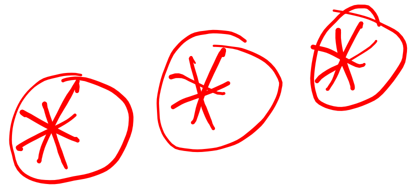
$\frac{\pi}{2} \leftarrow$ left-hand elliptical polarization, for $\varphi = \pi/2$ and A=B it is left-hand circular pol. (**LHCP**)

$\pi \leftarrow$ linear (LP)

$\frac{3\pi}{2} \leftarrow$ right-hand elliptical polarization, for $\varphi = 3\pi/2$ and A=B it is right-hand circular pol. (**RHCP**)

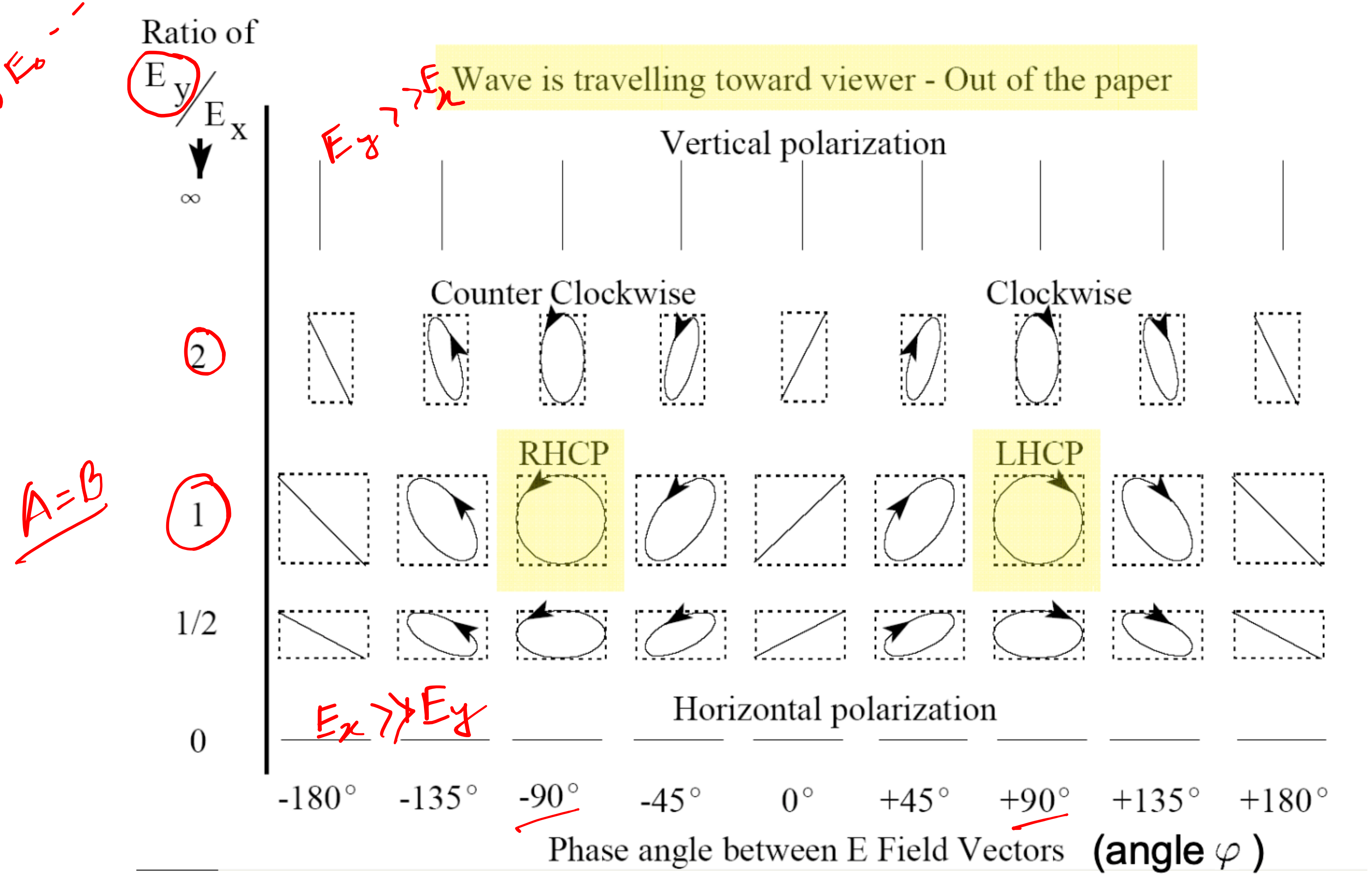
$2\pi \leftarrow$ linear (LP)

$\boxed{\varphi}$

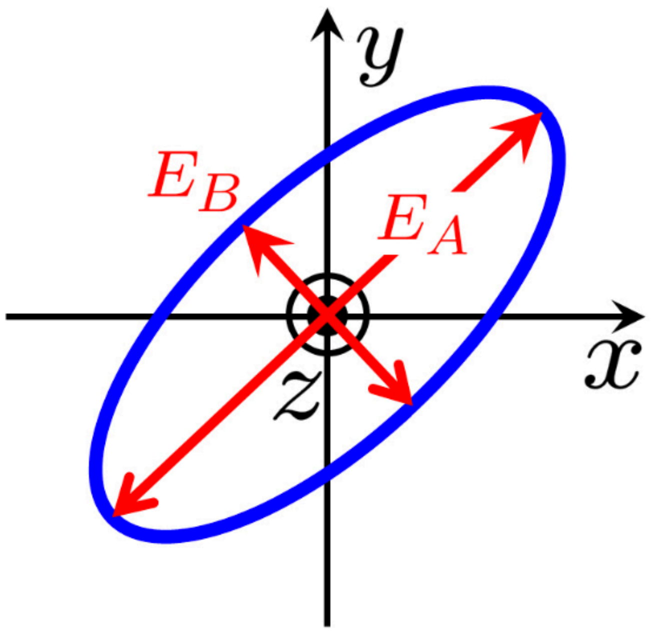


Polarization of EM Waves

$E_y = E_0$ —
$E_x = 0.5 E_0$ - -



Axial Ratio



$$AR = 20 \log \frac{E_A}{E_B}$$

major axis

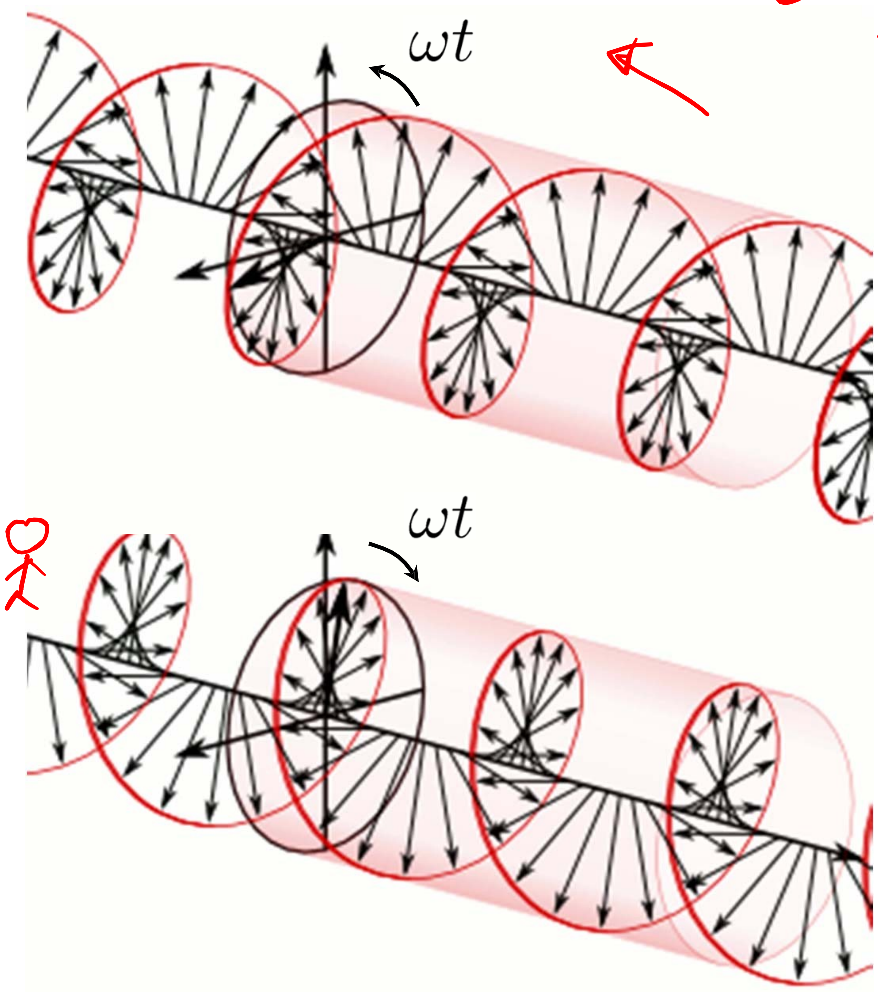
minor axis

Figure 4.9: Polarization ellipse and axial ratio

- Axial Ratio (AR) is the ratio of the main axes E_A and E_B of the non-perfect (i.e., non-circular) polarization ellipse.
- Typically, the AR is given in dB.
- Typical AR values are in a range of 0dB and 6dB.
- A linear polarization the axial ratio becomes infinity.

Circular Polarization

http://en.wikipedia.org/wiki/Circular_polarization



Helices can be either right-handed or left-handed. With the line of sight along the helix's axis, if a **clockwise screwing** motion moves the helix **away from the observer**, then it is called a **right-handed helix**; if towards the observer, then it is a left-handed helix.

counter clock

RHCP

(and spatial helix is left-handed)

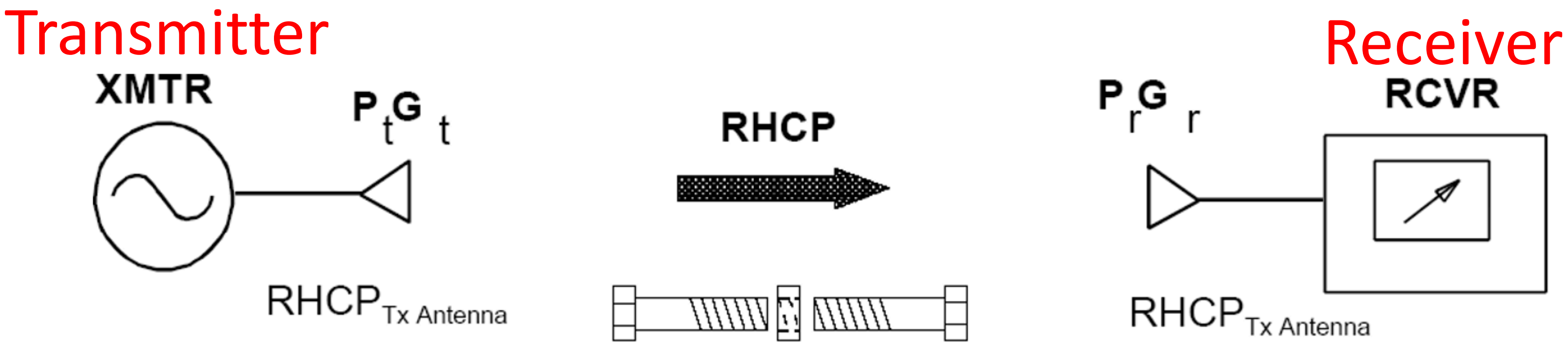
clockwise

LHCP

(and spatial helix is right-handed)

Sense of rotation is according to IEEE convention, .i.e., from the point of view of the wave's source.

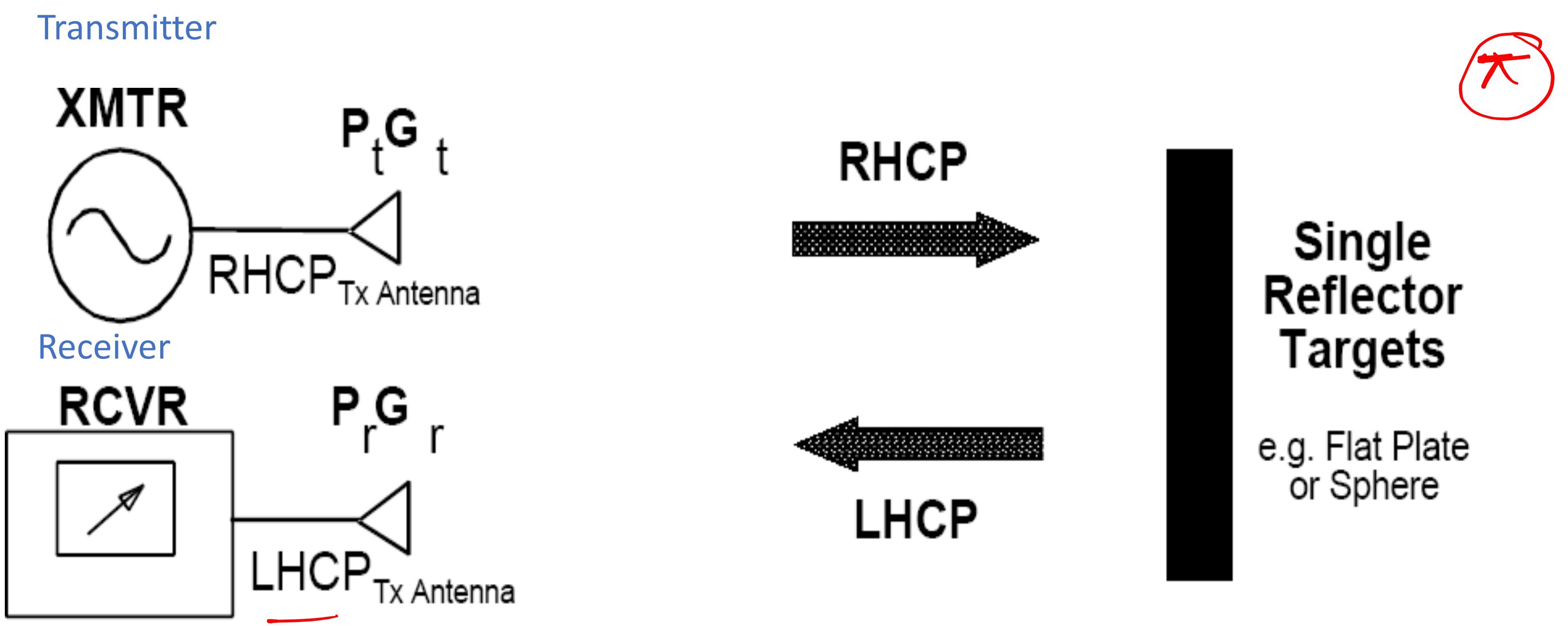
Polarization of EM Waves



NOTE: This figure depicts an example only, all polarizations can be reversed. In either case, <u>the antennas should be identical.</u>

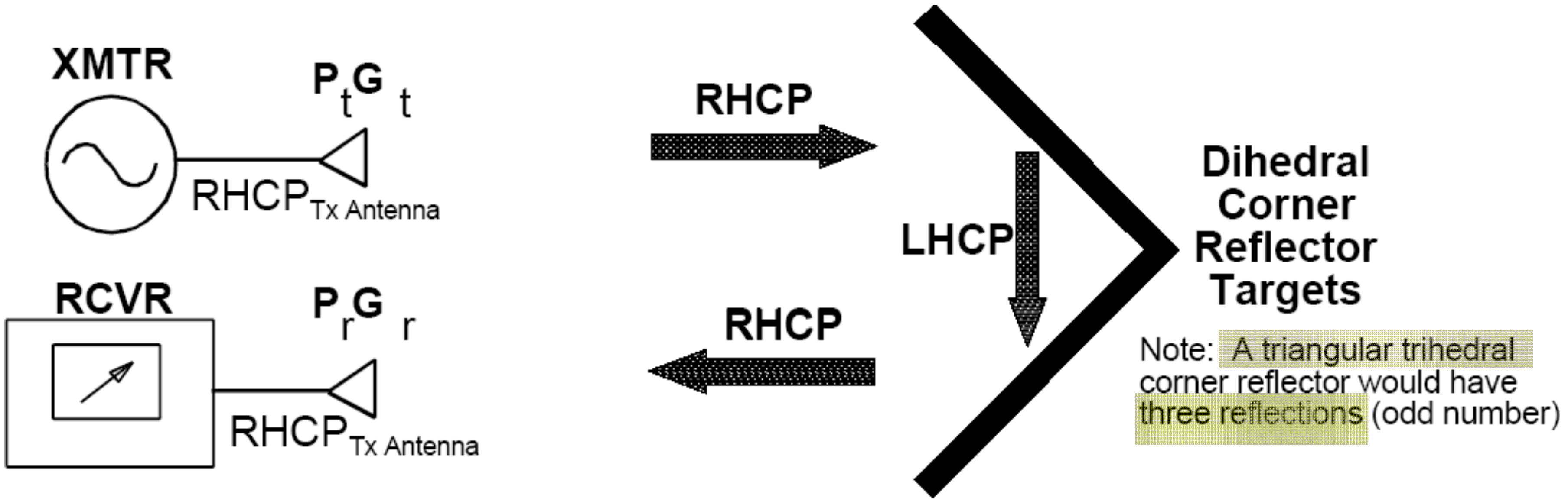
Wave propagation between two identical antennas is analogous to being able to thread a nut from one bolt to an identical opposite-facing bolt.

Polarization of EM Waves



NOTE: This figure depicts an example only, all polarizations can be reversed. In either case, <u>the antennas should have opposite polarization.</u>

Polarization of EM Waves



NOTE: This figure depicts an example only, all polarizations can be reversed.
In either case, the antennas should be identical.

Polarization of EM Waves

Table 1. Polarization Loss for Various Antenna Combinations

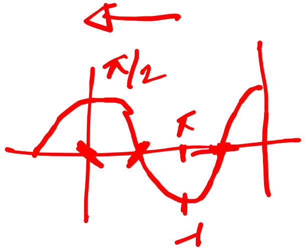
Transmit Antenna Polarization	Receive Antenna Polarization	Ratio of Power Received to Maximum Power					
		Theoretical		Practical Horn		Practical Spiral	
		Ratio in dB	as Ratio	Ratio in dB	as Ratio	Ratio in dB	as Ratio
Vertical	Vertical	0 dB	1	*	*	N/A	N/A
Vertical	Slant (45° or 135°)	-3 dB	½	*	*	N/A	N/A
Vertical	Horizontal	- ∞ dB	0	-20 dB	1/100	N/A	N/A
Vertical	Circular (right-hand or left-hand)	-3 dB	½	*	*	*	*
Horizontal	Horizontal	0 dB	1	*	*	N/A	N/A
Horizontal	Slant (45° or 135°)	-3 dB	½	*	*	N/A	N/A
Horizontal	Circular (right-hand or left-hand)	-3 dB	½	*	*	*	*
Circular (right-hand)	Circular (right-hand)	0 dB	1	*	*	*	*
Circular (right-hand)	Circular (left-hand)	- ∞ dB	0	-20 dB	1/100	-10 dB	1/10
Circular (right or left)	Slant (45° or 135°)	-3 dB	½	*	*	*	*

* Approximately the same as theoretical

$\cancel{\sin 0 = 0}$

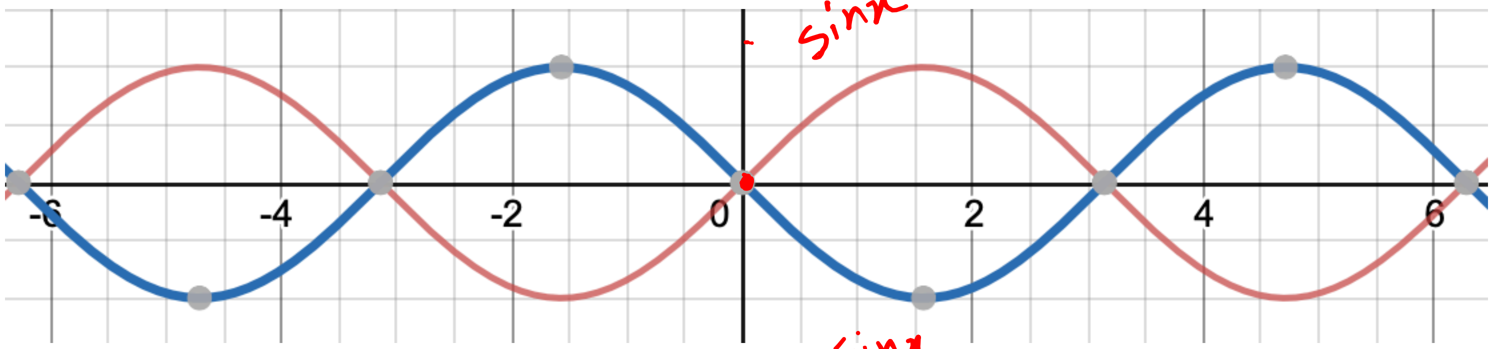
$\cancel{\cos 0 = 1}$

Graph Test



x

$\sin x$

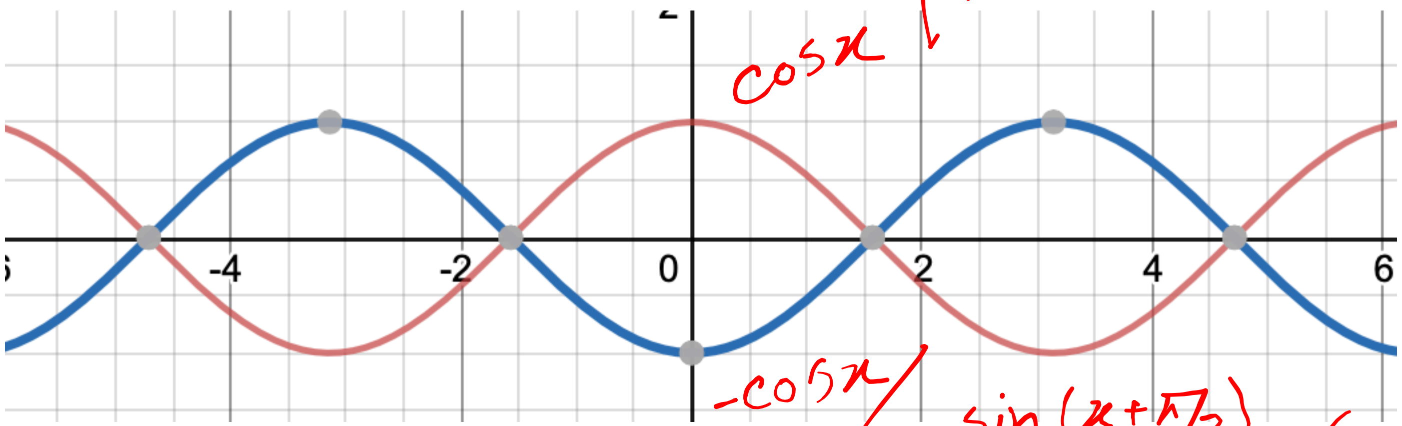


$-\sin x$

1. $\sin(x)$
2. $\cos\left(x + \frac{\pi}{2}\right)$ $(-\sin x)$

$\sin(x + \pi/2)$

$\cos x$

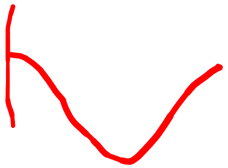


$-\cos x$

$-\sin(x + \pi/2)$

$\sin(x - \pi/2)$

1. $\cos(x)$
2. $\sin\left(x + \frac{3\pi}{2}\right)$



4.4 Problem 4

a) Determine the polarization of the following plane waves.

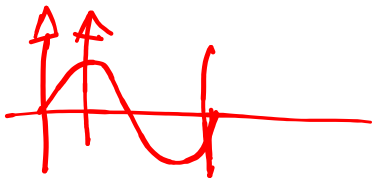
$$\begin{aligned}
\vec{E}_1 &= E_0 \cos(\omega t + kz)\,\vec{e}_x + E_0 \sin(\omega t + kz)\,\vec{e}_y \\
\vec{E}_2 &= E_0 \cos(\omega t + kz)\,\vec{e}_x - E_0 \sin(\omega t + kz)\,\vec{e}_y \\
\vec{E}_3 &= E_0 \cos(\omega t + kz)\,\vec{e}_x - 2E_0 \sin(\omega t + kz - \frac{\pi}{4})\,\vec{e}_y
\end{aligned}$$

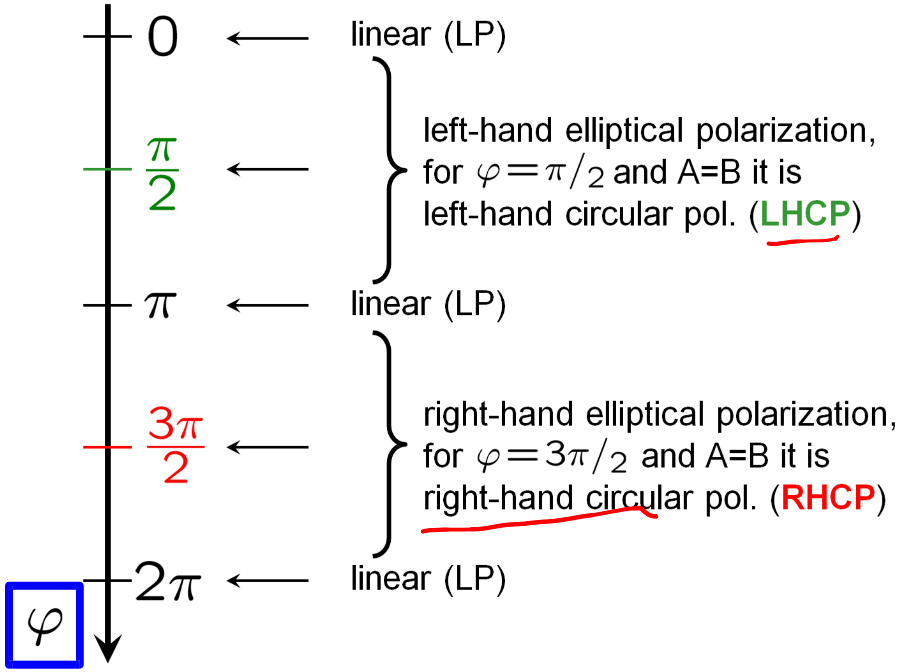
b) It is given the magnetic field intensity $\vec{H} = -H_1 \cos(\omega t - kz + \theta)\,\vec{e}_x + H_2 \cos(\omega t - kz)\,\vec{e}_y$. Show that the tip of the electric field vector may trace a line, a circle, or an ellipse over time in a fixed plane $z = const.$ Depict and rationalize your answers.

c) Show that a linearly polarized wave can be obtained as the superposition of two circularly polarized waves rotating in opposite directions but at the same angular rate.

$\cos(\omega t + kz + \pi/2)$

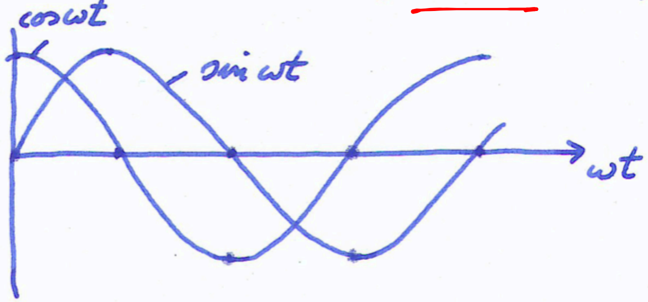
$$\boxed{\begin{aligned}
E_x &= A\cos(\omega t - k_z z) \\
E_y &= B\cos(\omega t - k_z z + \varphi)
\end{aligned}}$$

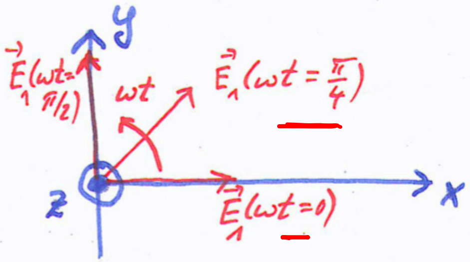




a) All waves $\vec{E}_1, \vec{E}_2, \vec{E}_3$ propagate in negative z-direction!

$\vec{E}_1 = E_0 \cos(\omega t + kz)\vec{e}_x + E_0 \sin(\omega t + kz)\vec{e}_y$





Because the wave propagates in negative z-direction its sense of polarization is left-handed ⇒ LHCP.

4.4 Problem 4

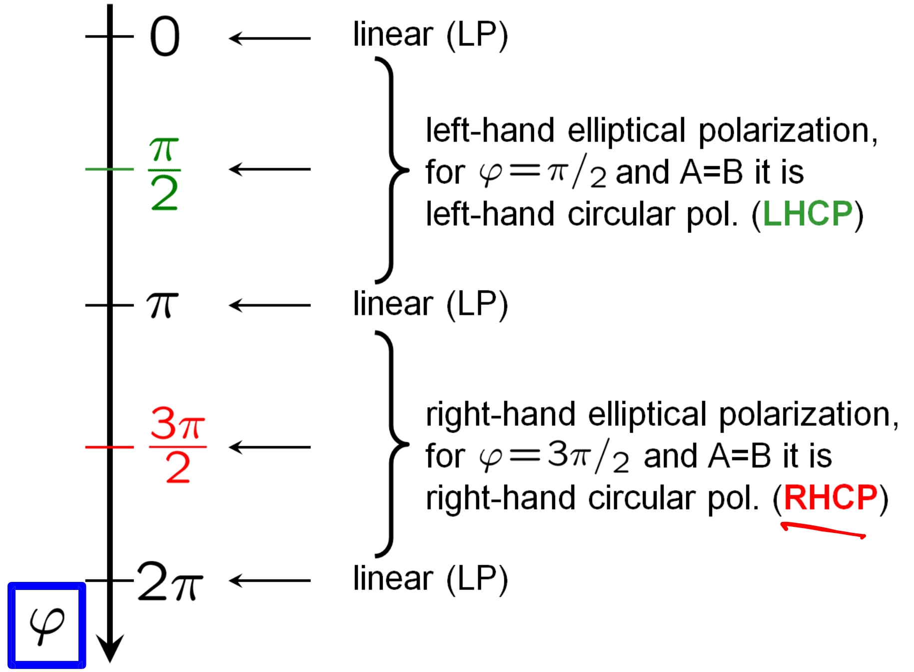
a) Determine the polarization of the following plane waves.

$$\vec{E}_1 = E_0 \cos(\omega t + kz)\,\vec{e}_x + E_0 \sin(\omega t + kz)\,\vec{e}_y$$
$$\vec{E}_2 = E_0 \cos(\omega t + kz)\,\vec{e}_x - E_0 \sin(\omega t + kz)\,\vec{e}_y$$
$$\vec{E}_3 = E_0 \cos(\omega t + kz)\,\vec{e}_x - 2E_0 \sin(\omega t + kz - \frac{\pi}{4})\,\vec{e}_y$$

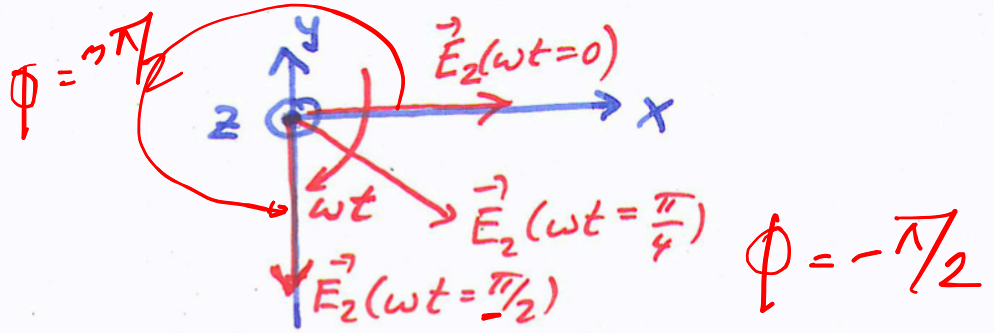
b) It is given the magnetic field intensity $\vec{H} = -H_1 \cos(\omega t - kz + \theta)\,\vec{e}_x + H_2 \cos(\omega t - kz)\,\vec{e}_y$. Show that the tip of the electric field vector may trace a line, a circle, or an ellipse over time in a fixed plane $z = const$. Depict and rationalize your answers.

c) Show that a linearly polarized wave can be obtained as the superposition of two circularly polarized waves rotating in opposite directions but at the same angular rate.

$$\boxed{\begin{aligned} E_x &= A\cos(\omega t - k_z z) \\ E_y &= B\cos(\omega t - k_z z + \varphi) \end{aligned}}$$



$$\vec{E}_2 = E_0 \cos(\omega t + kz)\vec{e}_x - E_0 \sin(\omega t + kz)\vec{e}_y$$



$\Rightarrow$ RHCP

4.4 Problem 4

a) Determine the polarization of the following plane waves.

$\pi + \pi/2 - \pi/4$

$$\vec{E}_1 = E_0 \cos(\omega t + kz)\,\vec{e}_x + E_0 \sin(\omega t + kz)\,\vec{e}_y$$
$$\vec{E}_2 = E_0 \cos(\omega t + kz)\,\vec{e}_x - E_0 \sin(\omega t + kz)\,\vec{e}_y$$
$$\vec{E}_3 = E_0 \cos(\omega t + kz)\,\vec{e}_x - 2E_0 \sin(\omega t + kz - \frac{\pi}{4})\,\vec{e}_y$$

b) It is given the magnetic field intensity $\vec{H} = -H_1 \cos(\omega t - kz + \theta)\,\vec{e}_x + H_2 \cos(\omega t - kz)\,\vec{e}_y$. Show that the tip of the electric field vector may trace a line, a circle, or an ellipse over time in a fixed plane $z = const$. Depict and rationalize your answers.

c) Show that a linearly polarized wave can be obtained as the superposition of two circularly polarized waves rotating in opposite directions but at the same angular rate.

$$\boxed{\begin{aligned} E_x &= A\cos(\omega t - k_z z) \\ E_y &= B\cos(\omega t - k_z z + \varphi) \end{aligned}}$$

$0 \leftarrow$ linear (LP)

left-hand elliptical polarization, for $\varphi = \pi/2$ and A=B it is left-hand circular pol. (**LHCP**)

$\frac{\pi}{2} \leftarrow$

$\pi \leftarrow$ linear (LP)

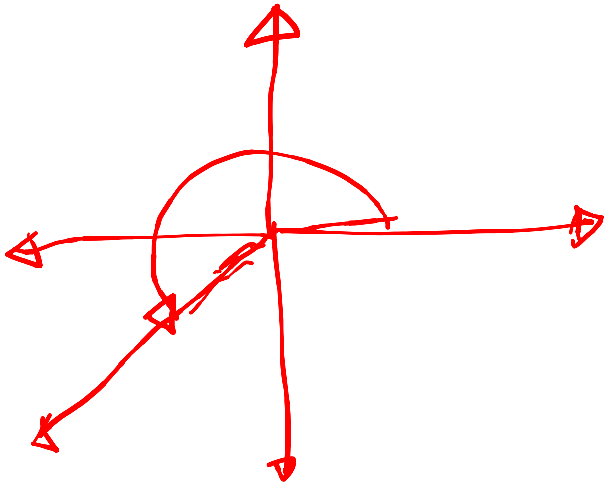
right-hand elliptical polarization, for $\varphi = 3\pi/2$ and A=B it is right-hand circular pol. (**RHCP**)

$\frac{3\pi}{2} \leftarrow$

$2\pi \leftarrow$ linear (LP)

$\boxed{\varphi}$

Handmade Graph!



4.4 Problem 4

a) Determine the polarization of the following plane waves.

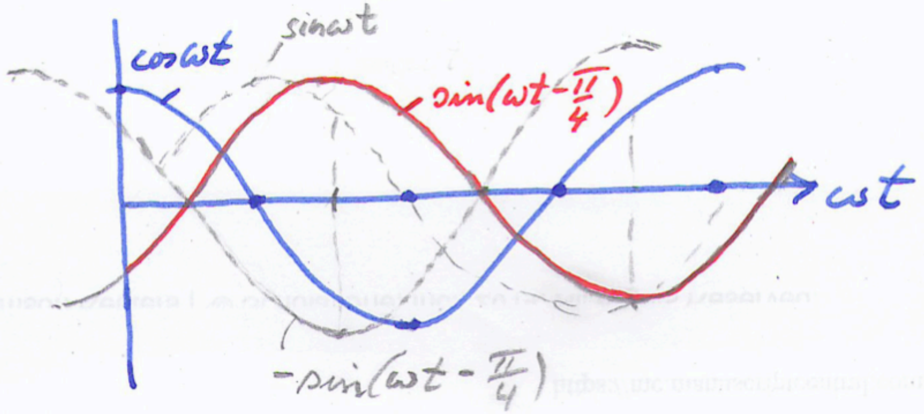
$$\vec{E}_1 = E_0 \cos(\omega t + kz)\,\vec{e}_x + E_0 \sin(\omega t + kz)\,\vec{e}_y$$
$$\vec{E}_2 = E_0 \cos(\omega t + kz)\,\vec{e}_x - E_0 \sin(\omega t + kz)\,\vec{e}_y$$
$$\vec{E}_3 = E_0 \cos(\omega t + kz)\,\vec{e}_x - 2E_0 \sin(\omega t + kz - \frac{\pi}{4})\,\vec{e}_y$$

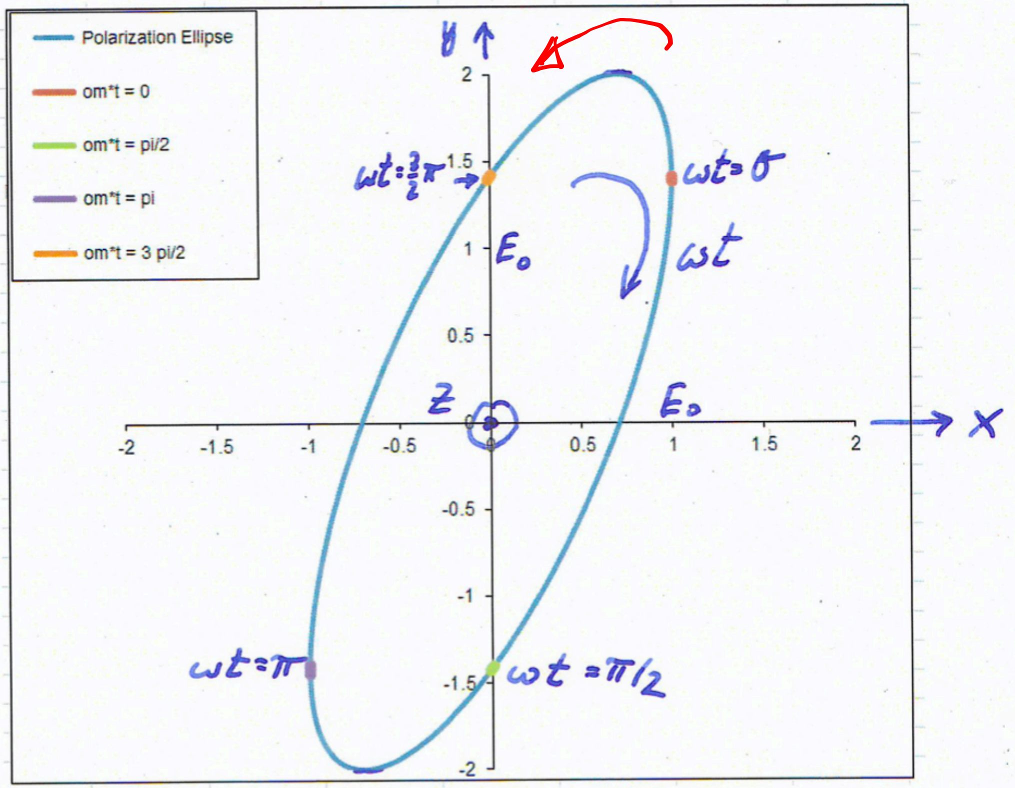
b) It is given the magnetic field intensity $\vec{H} = -H_1 \cos(\omega t - kz + \theta)\,\vec{e}_x + H_2 \cos(\omega t - kz)\,\vec{e}_y$. Show that the tip of the electric field vector may trace a line, a circle, or an ellipse over time in a fixed plane $z = const.$ Depict and rationalize your answers.

c) Show that a linearly polarized wave can be obtained as the superposition of two circularly polarized waves rotating in opposite directions but at the same angular rate.

$$AR = 20 \log \frac{E_A}{E_B}$$

$$\vec{E}_3 = E_0 \cos(\omega t + kz)\vec{e}_x - 2E_0 \sin(\omega t + kz - \frac{\pi}{4})\vec{e}_y$$





⇒ RHEP ; Axial Ratio = 10.17 dB

4.4 Problem 4

a) Determine the polarization of the following plane waves.

$$\vec{E}_1 = E_0\cos(\omega t + kz)\,\vec{e}_x + E_0\sin(\omega t + kz)\,\vec{e}_y$$
$$\vec{E}_2 = E_0\cos(\omega t + kz)\,\vec{e}_x - E_0\sin(\omega t + kz)\,\vec{e}_y$$
$$\vec{E}_3 = E_0\cos(\omega t + kz)\,\vec{e}_x - 2E_0\sin(\omega t + kz - \frac{\pi}{4})\,\vec{e}_y$$

E_y E_x

H_x H_y

b) It is given the magnetic field intensity $\vec{H} = -H_1\cos(\omega t - kz + \theta)\,\vec{e}_x + H_2\cos(\omega t - kz)\,\vec{e}_y$. Show that the tip of the electric field vector may trace a line, a circle, or an ellipse over time in a fixed plane $z = const$. Depict and rationalize your answers.

c) Show that a linearly polarized wave can be obtained as the superposition of two circularly polarized waves rotating in opposite directions but at the same angular rate.

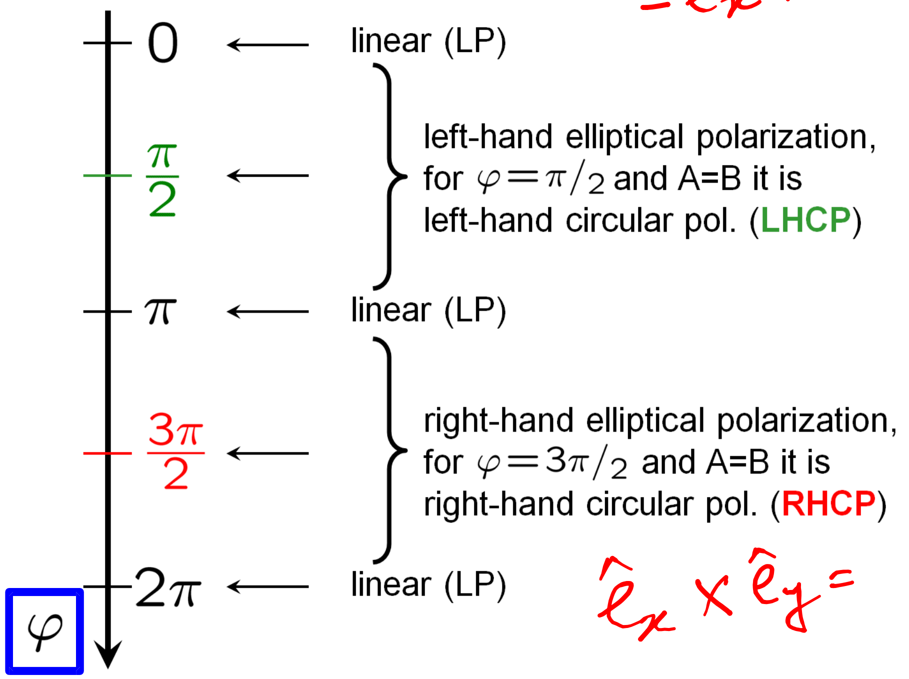
$$\boxed{\begin{aligned} E_x &= A\cos(\omega t - k_z z) \\ E_y &= B\cos(\omega t - k_z z + \varphi) \end{aligned}}$$

$\vec{E}\times\vec{H} = \vec{S}$

$E/H = Z_F$

$E = Z\,H$

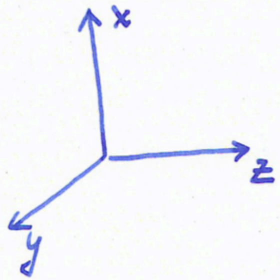
$-\hat{e}_x \times \hat{e}_z$



$\hat{e}_x \times \hat{e}_y = \hat{e}_z$

4.4b $\vec{H} = -H_1\cos(\omega t - kz + \Theta)\vec{e}_x + H_2\cos(\omega t - kz)\vec{e}_y$

The wave propagates in positive z-direction.



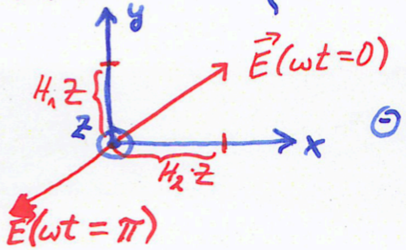
$\vec{S} = \vec{E}\times\vec{H} = |\vec{S}|\,\vec{e}_z$

$\Rightarrow \vec{H}\times\vec{e}_z$ is in parallel to $\vec{E}$

$\Rightarrow \vec{E} = Z\cdot(\vec{H}\times\vec{e}_z)$ with Z being the impedance of the wave.

$$\vec{E} = Z\begin{pmatrix} -H_1\cos(\omega t - kz + \Theta) \\ H_2\cos(\omega t - kz) \\ 0 \end{pmatrix} \times \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} = Z\cdot\begin{pmatrix} H_2\cos(\omega t - kz) \\ H_1\cos(\omega t - kz + \Theta) \\ 0 \end{pmatrix}$$

The tip of $\vec{E}$ traces a line if $\Theta = 0°$ or $\Theta = 180°$, i.e., linear polarisation



$\Theta = 0°$

4.4 Problem 4

a) Determine the polarization of the following plane waves.

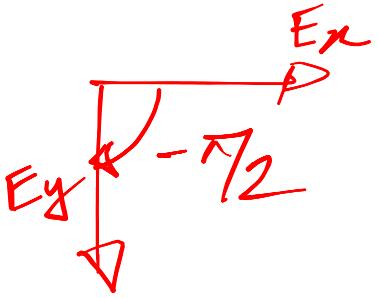
$$
\begin{aligned}
\vec{E}_1 &= E_0 \cos(\omega t + kz)\, \vec{e}_x + E_0 \sin(\omega t + kz)\, \vec{e}_y \\
\vec{E}_2 &= E_0 \cos(\omega t + kz)\, \vec{e}_x - E_0 \sin(\omega t + kz)\, \vec{e}_y \\
\vec{E}_3 &= E_0 \cos(\omega t + kz)\, \vec{e}_x - 2E_0 \sin(\omega t + kz - \frac{\pi}{4})\, \vec{e}_y
\end{aligned}
$$

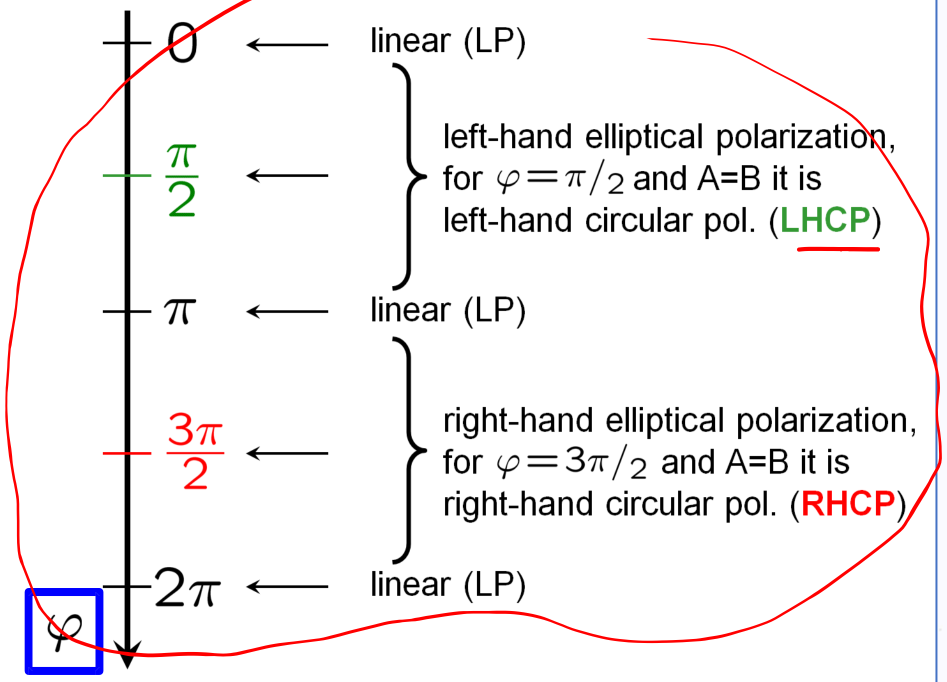
b) It is given the magnetic field intensity $\vec{H} = -H_1 \cos(\omega t - kz + \theta)\,\vec{e}_x + H_2 \cos(\omega t - kz)\,\vec{e}_y$. Show that the tip of the electric field vector may trace a line, a circle, or an ellipse over time in a fixed plane $z = const$. Depict and rationalize your answers.

c) Show that a linearly polarized wave can be obtained as the superposition of two circularly polarized waves rotating in opposite directions but at the same angular rate.

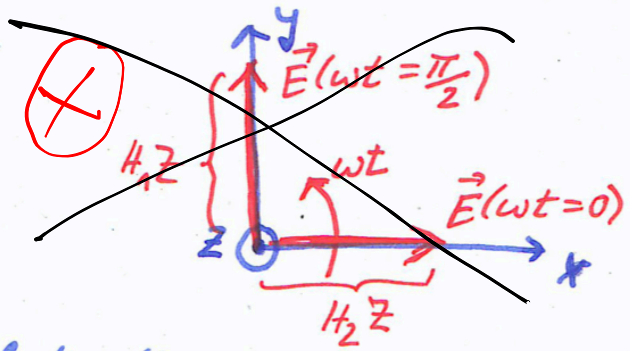
$$
\begin{aligned}
E_x &= A\cos(\omega t - k_z z) \\
E_y &= B\cos(\omega t - k_z z + \varphi)
\end{aligned}
$$

$-\pi/2$





$\Theta = -\frac{\pi}{2}$: $\cos(\omega t - kz - \frac{\pi}{2}) = \sin(\omega t - kz)$



$$\vec{E} = Z\begin{pmatrix} H_2 \cos(\omega t - kz) \\ H_1 \sin(\omega t - kz) \\ 0 \end{pmatrix}$$

if $H_1 = H_2 \Rightarrow$ RHCP (circular pol.)

if $H_1 \neq H_2 \Rightarrow$ RHEP (elliptical pol.)

4.4 Problem 4

a) Determine the polarization of the following plane waves.

$$
\begin{aligned}
\vec{E}_1 &= E_0\cos(\omega t + kz)\,\vec{e}_x + E_0\sin(\omega t + kz)\,\vec{e}_y\\
\vec{E}_2 &= E_0\cos(\omega t + kz)\,\vec{e}_x - E_0\sin(\omega t + kz)\,\vec{e}_y\\
\vec{E}_3 &= E_0\cos(\omega t + kz)\,\vec{e}_x - 2E_0\sin(\omega t + kz - \frac{\pi}{4})\,\vec{e}_y
\end{aligned}
$$

b) It is given the magnetic field intensity $\vec{H} = -H_1\cos(\omega t - kz + \theta)\,\vec{e}_x + H_2\cos(\omega t - kz)\,\vec{e}_y$. Show that the tip of the electric field vector may trace a line, a circle, or an ellipse over time in a fixed plane $z = const.$ Depict and rationalize your answers.

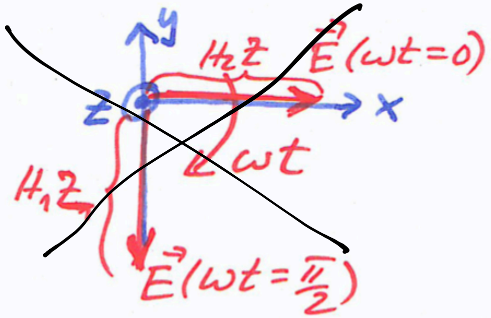
c) Show that a linearly polarized wave can be obtained as the superposition of two circularly polarized waves rotating in opposite directions but at the same angular rate.

$$
\boxed{
\begin{aligned}
E_x &= A\cos(\omega t - k_z z)\\
E_y &= B\cos(\omega t - k_z z + \varphi)
\end{aligned}
}
$$

$\boxed{\varphi}$

- 0 ⟵ linear (LP)
- $\frac{\pi}{2}$ ⟵ left-hand elliptical polarization, for $\varphi = \pi/2$ and A=B it is left-hand circular pol. (**LHCP**)
- π ⟵ linear (LP)
- $\frac{3\pi}{2}$ ⟵ right-hand elliptical polarization, for $\varphi = 3\pi/2$ and A=B it is right-hand circular pol. (**RHCP**)
- 2π ⟵ linear (LP)

$\theta = +\frac{\pi}{2}$: $\cos(\omega t - kz + \frac{\pi}{2}) = -\sin(\omega t - kz)$



$$\vec{E} = Z \cdot \begin{pmatrix} H_2\cos(\omega t - kz) \\ -H_1\sin(\omega t - kz) \\ 0 \end{pmatrix}$$

if $H_1 = H_2 \Rightarrow$ LHCP (circular pol.)

if $H_1 \neq H_2 \Rightarrow$ LHEP (elliptical pol.)

for any other θ: elliptical polarization, even for $H_1 = H_2$!

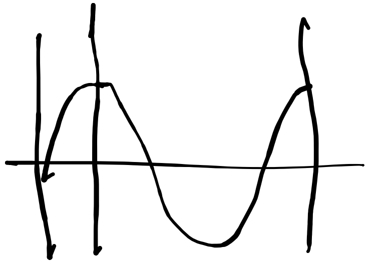
4.4 Problem 4

a) Determine the polarization of the following plane waves.

$$
\begin{aligned}
\vec{E}_1 &= E_0\cos(\omega t + kz)\,\vec{e}_x + E_0\sin(\omega t + kz)\,\vec{e}_y\\
\vec{E}_2 &= E_0\cos(\omega t + kz)\,\vec{e}_x - E_0\sin(\omega t + kz)\,\vec{e}_y\\
\vec{E}_3 &= E_0\cos(\omega t + kz)\,\vec{e}_x - 2E_0\sin(\omega t + kz - \frac{\pi}{4})\,\vec{e}_y
\end{aligned}
$$

b) It is given the magnetic field intensity $\vec{H} = -H_1\cos(\omega t - kz + \theta)\,\vec{e}_x + H_2\cos(\omega t - kz)\,\vec{e}_y$. Show that the tip of the electric field vector may trace a line, a circle, or an ellipse over time in a fixed plane $z = const.$ Depict and rationalize your answers.

c) Show that a linearly polarized wave can be obtained as the superposition of two circularly polarized waves rotating in opposite directions but at the same angular rate.



$E_x = 2E_0\cos(\omega t - kz)$

LHCP + RHCP

4.4c

$$\vec{E}_1 = E_0\underbrace{\begin{pmatrix}\cos(\omega t - kz)\\ \cos(\omega t - kz - \pi/2)\\ 0\end{pmatrix}}_{RHCP} \;;\quad \vec{E}_2 = E_0\underbrace{\begin{pmatrix}\cos(\omega t - kz)\\ \cos(\omega t - kz + \frac{\pi}{2})\\ 0\end{pmatrix}}_{LHCP}$$

$$\vec{E} = \vec{E}_1 + \vec{E}_2 = E_0\begin{pmatrix}2\cos(\omega t - kz)\\ \cos(\omega t - kz - \frac{\pi}{2}) + \cos(\omega t - kz + \frac{\pi}{2})\\ 0\end{pmatrix}$$

$$= E_0\begin{pmatrix}2\cos(\omega t - kz)\\ \cancel{\sin(\omega t - kz)} - \cancel{\sin(\omega t - kz)}\\ 0\end{pmatrix} = 2E_0\begin{pmatrix}\cos(\omega t - kz)\\ 0\\ 0\end{pmatrix}$$

i.e., LP